HAMILTON'S INTERMEDIATE ARITHMETIC

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INTERMEDIATE ARITHMETIC

FOR GRADED SCHOOLS

BY

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PREFACE

THIS Intermediate Arithmetic is intended for fifth and sixth years. It is divided into two parts.

Part One includes the work of the fifth year. It gives an elementary treatment of Fractions, Decimals, Percentage, Denominate Numbers, and Practical Measurements.

Part Two includes the work of the sixth year. It gives a complete treatment of Fractions, Decimals, Denominate Numbers, and an elementary course in Percentage, Practical Measurements, and business forms.

The aim of this course is twofold: first, to give the pupil mathematical skill; second, to give him mathematical power.

To these ends attention is invited to the following:

1. The prominence given to drill intended to give skill.
2. The "Study of Problems" intended to give mathematical power.
3. The plan which provides an easy treatment of each subject before the complete treatment.
4. The easy steps in gradation.
5. The emphasis given to business arithmetic.
6. The abundance of exercises for oral drill.

The importance of *oral drill* has led the author to lay special emphasis on this kind of work. Pupils should be drilled thoroughly on the oral development and exercises in each subject, before taking up the written work.

The two treatments of topics, the number and the variety of problems, the systematic reviews, and the easy steps in the gradation of the work will, we believe, meet with the approval of teachers.

SAMUEL HAMILTON.

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FIFTH YEAR

SIXTH YEAR

INTERMEDIATE ARITHMETIC

PART I — FIFTH YEAR

REVIEW OF NOTATION AND NUMERATION

1. In 364 for what does the 4 stand? the 6? the 3? 364 = 3 hundreds, 6 tens, 4 ones.

2. Tell for what each figure stands in 2475. Read the number.

Each figure has a value that depends upon its **place** in the number. The first place beginning at the right of a number is called **ones'** place; the second, **tens'** place; the third, **hundreds'** place; the fourth, **thousands'** place, etc.

3. Show that the left-hand 2 in the number 222 has a value, because of its place, that is 10 times the value of the middle 2, and 100 times the value of the right-hand 2.

When a number contains more than three figures, it is separated into groups or **periods** of three figures each, beginning at the right. The names of the places in each period are the same. The name of the first period is **units**; of the second period, **thousands**; of the third, **millions**; and of the fourth, **billions**.

4. In the number 864,203,614,869, what name is given to the period 864? to the period 203? to 614? to 869?

Notation is a method of writing numbers by means of symbols.

Numeration is a method of reading numbers represented by symbols.

Beginning at the right, mark off into periods, and then read:

5. 2439	8. 75865	11. 405682	14. 15785639
6. 3750	9. 775570	12. 1405600	15. 155689375
7. 4005	10. 205605	13. 6850000	16. 100370400

Observe that: *naughts* are enumerated but never read; that *and* is never read between integers; and that the *first period* is simply read but not named, — thus 375 is read three hundred seventy-five.

Write:

17. 6 thousand 3 hundred 75.

18. 10 thousand 4 hundred 6.

19. 150 thousand 5 hundred 25.

20. 5 million 825 thousand 5 hundred 4.

21. 205 million 25 thousand 3.

22. 200 million 22 thousand 60.

23. 5 billion 600 million 27 thousand 9.

24. 105 million 5 thousand 5.

25. 8 hundred million 8 thousand 85.

26. Ten thousand ten hundred ten.

27. Five hundred million two hundred fifty.

28. Six million six thousand six.

29. Sixty billion one hundred one million one thousand five hundred seventy-nine.

30. Forty million four.

31. 8 hundred million 8 thousand one hundred.

32. Ninety million six hundred thousand nine hundred two.

33. Eight hundred million two hundred twenty-eight thousand two hundred one.

34. One billion nine hundred million one thousand nine hundred fifty-four.

Roman Notation

The seven letters used in Roman notation are:

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

The other numbers are represented by combinations thus:

I. *When a letter is followed by the same letter or by one of less value, the values of the letters are to be added:* Thus XX represents 20; XI represents 11.

II. *When a letter is followed by one of greater value, the value of the smaller is to be subtracted from that of the greater.* Thus IV represents 4; IX represents 9.

III. *When a letter is placed between two letters of greater value, the value of the smaller is to be subtracted from the sum of the other two.* Thus XIV represents 14; XIX represents 19.

A bar placed over a letter multiplies its value by 1000. Thus $\overline{\text{V}}$ represents 5000.

The following table further illustrates the system.

I, 1	VIII, 8	XVI, 16	LXXX, 80	DCC, 700
II, 2	IX, 9	XX, 20	XC, 90	DCCC, 800
III, 3	X, 10	XXX, 30	C, 100	CM, 900
IV, 4	XI, 11	XL, 40	CCC, 300	M, 1000
V, 5	XII, 12	L, 50	CD, 400	MCM, 1900
VI, 6	XIV, 14	LX, 60	D, 500	$\overline{\text{V}}$, 5000
VII, 7	XV, 15	LXX, 70	DC, 600	$\overline{\text{M}}$, 1,000,000

Read:

1.	XXI	LXV	CDXLIV	$\overline{\text{D}}$L	$\overline{\text{MV}}$	XCIX
2.	XXIV	LXXI	MCMVII	$\overline{\text{M}}$D	DCCI	$\overline{\text{C}}$XL

3. Express in Roman notation: 8, 25, 39, 43, 52, 67, 79, 86, 94, 107, 119, 125, 408, 525, 693, 709, 844, 915, 1040, 1906, 2645, 5074, 10000.

Dollars and Cents

In writing dollars and cents place a period, called a **decimal point**, after dollars and before cents. Thus, $8.55 is read 8 dollars and 55 cents. In writing cents alone the dollar sign must precede the decimal point. Thus, 25 cents is written $.25 or $0.25; 5 cents is written $.05 or $0.05.

1. Read: $.05; $.25; $0.07; $6.05; $7.09; $4.72.

2. Read, then change to cents. Thus, $3.85 = 385¢. $2.05; $.70; $0.05; $7.09; $8.00; $.75; $3.50.

3. Change to dollars and cents: 55¢; 85¢; 870¢; 1002¢.

4. Write: eighty-five cents; nine dollars two cents; twenty-two dollars; nine hundred dollars six cents.

REVIEW OF ADDITION

Addition is the process of finding a number that equals two or more other numbers.

The **addends** are the numbers to be added.

The **sum** or **amount** is the result of addition.

Drill thoroughly on these and similar combinations:

1.	2	13	7	15	14	12	9	15	15	12	13	7
	7	8	3	9	6	5	8	7	5	4	4	7
2.	6	18	14	11	11	11	5	6	11	13	12	4
	15	8	4	1	3	9	8	10	4	9	8	7
3.	12	13	14	6	11	5	6	11	9	19	6	12
	6	3	5	7	5	13	8	6	9	7	13	9
4.	7	16	8	2	9	11	4	12	6	17	14	17
	9	6	11	12	13	2	9	13	19	8	8	7

Rapid counting by 2's to 9's should be practiced daily.

Add: **5.**

462 } 8
376 }
814 } 10
296 }

6.

321
456 } 15
289 }
315

7.

287 } 10
823 }
541 } 10
569 }

8.

463
726 } 15
339 }
187

9.

268 } 12
374 }
827 } 10
693 }

To **test** addition, add from the bottom up; then from the top down.

10. Add 4613, 892, 812, 928.

4613 } 5
892 }
812 } 10
928 }
7245

Think: 10, 15. Write 5 and carry 1. 3, 13, 14. Write 4 and carry 1. 10, 26, 32. Write 2 and carry 3. 7. Sum, 7245.

Add the columns; then add across from left to right.

11.	**12.**	**13.**	**14.**	**15.**	TOTALS
243	629	928	872	745	
638	483	465	936	367	
729	948	738	148	258	
643	629	624	567	189	

16.	**17.**	**18.**	**19.**	TOTALS
$248	$296	$456.25	$415.05	
729	938	273.43	586.72	
361	695	816.32	293.33	
284	786	928.29	718.23	
816	249	623.18	826.34	

Like numbers are numbers that express the same kind of units, as 3 feet and 5 feet or 3 and 5; but not 3 feet and 5 cents.

Only like numbers can be added or subtracted.

REVIEW OF SUBTRACTION

Subtraction is the process of finding the difference between two numbers or of taking one number from another.

The **minuend** is the greater number; the **subtrahend** is the smaller number; and the **difference** or **remainder** is the answer.

Drill thoroughly on these combinations in subtraction:

1.	9	15	12	16	11	13	12	9	8	17	15	11
	4	7	6	7	4	9	3	3	2	8	4	3
2.	7	10	10	17	18	13	15	16	13	14	17	15
	2	7	6	9	9	7	6	9	8	9	5	9
3.	11	16	15	12	8	9	13	10	11	14	12	14
	9	8	8	5	3	6	6	4	6	7	4	6
4.	15	13	11	10	15	14	18	13	17	13	15	16
	11	4	8	9	5	8	16	11	13	12	10	14

5. A grocer sells butter for 18¢ and receives a half dollar. In making change he adds 2 cents to 18 cents, and then adds the pieces of money that will make 50 cents. What are they?

6. Make change from $1 for a purchase of 72¢; 85¢; 64¢.

This is called the **adding** or the **making change** method of subtraction.

7. Solve by this method: 842 − 385.

842
385
457

Think: What number added to 5 will make 12? (**7.**) Write down 7; carry one. What number added to 9 (8 + 1) will make 14? (**5.**) Write down 5; carry 1. What number added to 4 (3 + 1) will make 8? (**4.**) Write down 4.

To **test** subtraction, add the remainder and the subtrahend.

Find the difference between:

8.	44	69	213	761	298	723	842	841
	39	27	124	148	149	476	364	563

9. 36005 – 19096
10. 90000 – 27938
11. 23000 – 17500
12. $4629.70 – $3675.84
13. $1475.558 – $539.47
14. $3000.73 – $2036.75
15. $9143.65 – $6183.69
16. $24000.47 – $6937.64
17. $2039.05 – $1729.89
18. $9400.37 – $2869.94
19. 630209 – 189768
20. 620005 – 246937
21. 610034 – 263805
22. $2473.87 – $629.75
23. $9000.45 – $4167.23
24. $6343.75 – $900.84
25. $9143.92 – $6287.75
26. $4816.75 – $2407.84
27. $94367.48 – $21697.83
28. $21485.86 – $11475.97

REVIEW OF MULTIPLICATION

Multiplication is the process of taking one number as many times as there are units in another.

The **multiplicand** is the number multiplied; the **multiplier** is the number showing how many times the multiplicand is taken; and the **product** is the result of multiplication.

1. $6+6+6+6=?$ In what short way can you find the answer? Multiplication is a *shortened* form of *addition*.

2. $3 \times \$7 = \$21$. What number is multiplied? By what number is it multiplied? What is the name of the answer?

3. Read: 5×4 ft.; 6×3 doz.; 9×2 yd.; $8 \times 7¢$.

When the multiplier is written *before* the multiplication sign, $\times$, as in 4×8 bu., it is read "*times*" as 4 times 8 bu.

When the multiplier is written *after* the sign, it is read "*multiplied by*." Thus, 4 ft. × 8 is read 4 ft. multiplied by 8.

4. Does 6 apples times 4 make sense? The multiplier cannot name things; as, feet, hours, etc., since it indicates the number of times the multiplicand is to be taken.

An **abstract number** is a number used without reference to a particular thing; as, 5, 6, 50.

A **concrete number** is a number used with reference to a particular thing; as, 5 trees, 6 dollars.

The multiplier is always regarded as an *abstract number* since it shows how many times the multiplicand is taken. The product is always like the multiplicand.

5. Select the multipliers and the multiplicands:

6×9 doz. 3×9 qt. 8 hr. $\times$ 5.

6. Find the cost of 15 oranges at 3¢ each.

7. How many minutes are there in 4 hours? in 8 hours?

8. Multiply 323 bu. by 25.

Multiplicand	323 bu.		323 bu.
Multiplier	25		25
1st partial product	1615 bu.	$= 5 \times 323$ bu.	1615
2d partial product	6460 bu.	$= 20 \times 323$ bu.	646
Entire product	8075 bu.	$= 25 \times 323$ bu.	8075 bu.

To **test** multiplication, use the multiplicand for a multiplier and perform the multiplication again; or divide the product by the multiplier.

9. How much will 48 chairs cost at $1.25 each?

$1.25
48
1000
500
$60.00

Multiply as before, and mark off as many cents in the product as you have in the multiplicand.

10. 45×63 bu. $= ?$
11. 29×87 ft. $= ?$
12. 46×215 doz. $= ?$
13. $78 \times \$3.26$
14. $86 \times \$2.93$
15. $91 \times \$1.45$

16.
$$\begin{array}{r} 85 \\ 670 \\ \hline 5950 \\ 510 \\ \hline 56950 \end{array}$$

17.
$$\begin{array}{r} 6754 \\ 608 \\ \hline 54032 \\ 40524 \\ \hline 4106432 \end{array}$$

18.
$$\begin{array}{r} 6070 \\ 8006 \\ \hline 36420 \\ 48560 \\ \hline 48596420 \end{array}$$

Multiply and test:

19. 809×3750
20. 370×2009
21. 400×3098
22. 209×6708
23. 609×8078
24. 458×6009
25. 650×8079
26. 407×7900
27. 608×8004
28. 440×7980
29. 7605×8798
30. 5008×2347
31. 2705×8750
32. 3075×2005
33. 4706×2609
34. 8005×3672
35. 6723×4980
36. 2050×2006

FACTORS AND MULTIPLES

1. Supply the missing numbers:

$6 \times 4 = ?$	$2 \times 5 = ?$	$? \times 2 = 4$
$? \times 3 = 9$	$? \times 7 = 21$	$? \times 3 = 27$
$8 \times ? = 48$	$? \times 9 = 54$	$7 \times ? = 63$

2. The multiplicand and the multiplier are called **factors** of the product. 3 is one factor of 12. What is the other factor?

The product divided by one factor gives the other factor.

3. The product is 84 and one factor is 12. What is the other factor?

4. Find the unknown factors: product 96, one factor 8; product 144, one factor 12; product 284, one factor 4; product 1080, one factor 9; product 1274, one factor 7.

An **integer** or an **integral number** is a whole number.

A **multiple** of a number is one or more integral times the number.

5. 28 is a multiple of 4 and 7. 22 is a multiple of what numbers? 36 is a multiple of what two numbers?

6. Name all the multiples of 2 from 2 to 50.

7. Name all the multiples of 3 from 3 to 66.

8. Name two multiples of 4 and 6; of 8 and 10.

9. Tell which of the following are multiples of 3; of 5; of 9:

45 63 72 65 108 24 90 18 27

REVIEW OF DIVISION

Division is the process of finding how many times one number contains another, or of separating a number into equal parts.

The **dividend** is the number divided; the **divisor** is the number by which we divide; and the **quotient** is the result of division.

The **remainder** is the part of the dividend remaining when the division is not exact.

1. How many times is \$10 contained in \$120? Here both dividend and divisor are concrete numbers. Are they alike? The quotient, 12, is abstract because the \$120 is measured or divided by \$10; or \$10 is contained 12 times in \$120.

2. In the problem \$120 ÷ 10 = \$12, or $\frac{1}{10}$ of \$120 = \$12, the divisor is abstract. Observe that the quotient is like the dividend, and that \$120 is separated into 10 equal *parts*. This kind of division is called **partition**.

Division may be indicated in 3 ways: thus $376 \div 8$, or $8\overline{)376}$ or $\frac{376}{8}$.

Find quotients:

3.	$522 \div 6$	8.	$132¢ \div 12¢$	13.	$12\overline{)\$288}$
4.	$408 \div 12$	9.	144 oz. $\div$ 16 oz.	14.	$9\overline{)333}$ ft.
5.	$801 \div 9$	10.	147 wk. $\div$ 7 wk.	15.	$6\overline{)384}$ qt.
6.	$686 \div 7$	11.	$\$217 \div \$7$	16.	$8\overline{)504}$ lb.
7.	$\frac{301}{7}$	12.	$\frac{1054 \text{ hr.}}{24 \text{ hr.}}$	17.	$\frac{984 \text{ ft.}}{8 \text{ ft.}}$

18. Divide 81906 by 34.

$$\begin{array}{r} 2409 \quad \text{Quotient} \\ 34\overline{)81906} \quad \text{Dividend} \\ 68 \\ \hline 139 \\ 136 \\ \hline 306 \\ 306 \end{array}$$

Notice in the third division that 30 is smaller than the divisor. Place naught in the quotient, write 6 to the right of 30, and divide 306 by 34.

To test division, multiply the quotient by the divisor. If there is a remainder, add it to the product.

Divide and test:

19.	218645 $\div$ 44	26.	95625 by 43	33.	70004 by 172
20.	218465 $\div$ 95	27.	62181 by 63	34.	74029 by 181
21.	836219 $\div$ 23	28.	63595 by 79	35.	26686 by 378
22.	346924 $\div$ 63	29.	736840 by 65	36.	441324 by 492
23.	637185 $\div$ 88	30.	406090 by 65	37.	407912 by 578
24.	46221 $\div$ 21	31.	13965 by 133	38.	635973 by 709
25.	28497 $\div$ 21	32.	16023 by 147	39.	720044 by 908

Find:

40. $\frac{1}{25}$ of 2625 42. $\frac{1}{54}$ of \$29322 44. $\frac{1}{105}$ of 10290
41. $\frac{1}{36}$ of 14652 43. $\frac{1}{84}$ of 76104 45. $\frac{1}{96}$ of 35904

Multiplying and dividing by 10, 20, 100, etc.

1. Multiply 42 by 10. Annex a naught to the right of 42. Is there any difference in the products? Which of these two methods of multiplying is the *shorter?* Multiply 42 by 100. Annex two naughts to the right of 42.

2. Multiply each of the following numbers by 10; by 100:

43 26 75 96 283 694 786 813 465 710

3. Divide 40 by 10. Cut off a naught from the right of 40. Is there any difference in the quotients? Which of these two methods of dividing is the *shorter?*

4. Divide each of the following numbers by 10; by 100:

300 4600 27300 61900 81400 86800 20000

5. Divide 2436 by 100. $1|00\,\overline{)}\,24|36$ $24\frac{36}{100}$

6. $8600 \div 10$ 8. $814256 \div 600$ 10. $378000 \div 6000$
7. $46153 \div 100$ 9. $384269 \div 600$ 11. $9285000 \div 11000$

Division of dollars and cents.

1. If 20 bushels of apples cost \$18.00, find the cost per bushel.

\$0.90 cost of 1 bushel.
$20\overline{)18.00}$.

2. At 8¢ per basket how many baskets of berries can be purchased for \$1.84?

23 no. of baskets.
$8¢\overline{)184¢}$

Find the cost of 1 if:

3. 12 yd. cloth cost \$13.44.
4. 16 books cost \$13.60.
5. 15 yd. ribbon cost \$1.35.

Find the number if:

6. Pads at 8¢ cost \$3.20.
7. Caps at 72¢ cost \$10.80.
8. Plates at 18¢ cost \$8.10.

TEST EXERCISES

1. I bought 46 lb. of sugar @ 7¢, 95 lb. of coffee @ 23¢, and 73 doz. eggs @ 18¢. Find the cost of all.

2. James had $24.36 in a bank and drew $17.49 out. How much had he left in the bank?

3. How many books @ $.34 can be bought for $15.30?

4. If 54 cows cost $4698, how much does each cost?

5. At 25¢ each, how many caps can be bought for $149?

6. If a school requires 497 pads of paper for one month, how many will be needed for 3 years of 10 months each?

7. If 43 barrels of cement cost $208.55, how much will 59 barrels cost?

8. How much will 96 quarts of cranberries cost, if 24 quarts cost $3.60?

9. A western farmer raised 6741 bu. of oats. He kept 349 bu. for feed, and sold the remainder at $.65 a bushel. How much did he receive?

10. A merchant sold 32 yd. of cloth at 12¢ a yard, and 192 yd. at 8¢ a yard. How much did he receive?

11. At 4¢ a pint, find the cost of 6 gal. 1 pt. of milk.

12. At 8¢ a quart, find the cost of 4 bu. 2 pk. of beans.

13. At 4¢ an ounce, find the cost of 3 lb. 3 oz. of ginger.

14. At 24¢ a pound, find the cost of 29 lb. of butter.

15. Find the cost of sending a 15 word telegram from New York to Denver, at 75¢ for the first ten words and 5¢ for each additional word.

16. How much will be saved by sending the same message at night, the night rate being 60¢ for the first ten words and 4¢ for each additional word?

17. A telephone message from New York to Boston costs $1.25 for the first three minutes and $.40 for each additional minute. Find the cost of talking 17 minutes.

18. A grocer bought 57 crates of berries at $4.75 a crate. If he sells them at $5.15 a crate, how much will he gain?

19. How much will a fruit dealer gain if he buys 174 boxes of oranges for $739.50, and sells them at $5 a box?

20. A merchant bought 136 barrels of apples at $1.35 a barrel; but 30 barrels were damaged. If he sold the remainder at $1.75 a barrel, did he gain or lose and how much?

21. A man bought 36 gallons of milk for $7.20. He sold it for 35¢ a gallon. How much did he gain?

22. A merchant bought 800 bushels of potatoes at 45¢ a bushel. He sold one half of them at 60¢ a bushel, and the remainder at 40¢ a bushel. Did he gain or lose, and how much?

23. Find the perimeter or distance around a room 18 ft. by 15 ft.

24. At 25¢ a square yard, how much will it cost to paint the floor of the room?

25. Mr. Adams worked 8 hours a day for 26 days at $.37 an hour. How much did he earn?

26. Find the entire cost of:

6 yd. silk @ 85¢.
5 bu. peaches @ $2.25.
2 doz. boxes soap @ $4.75.

27. Make out a bill for:

9 T. hard coal @ $6.75.
12 T. soft coal @ $3.75.
15 cwt. sugar @ 5¢.

28. A merchant buys from a farmer:

25 bu. of corn @ 45¢.
36 doz. eggs @ 23¢.
16 bu. apples @ 75¢.

The farmer buys from the merchant:

6 brooms @ 25¢.
22 yd. of carpet @ 90¢.
6 chairs @ $2.50.

Which person owes the other, and how much?

29. In a factory there are 56 men employed at $2.25 a day; 12 men at $3.75 a day; 25 boys at $.87 a day; 8 women at $1.75 a day. The other expenses are $267 a day. How much does it cost to keep the factory going a month of 26 days?

30. $748 was paid in wages to some workmen, giving each man $1.87. How many workmen were there?

31. How many square feet are there in a field 200 feet wide and 12 rods long?

32. How many feet is it around the field?

33. A boy sold 12 gallons 2 quarts of milk at 8¢ a quart. How much did he receive for it?

34. How much will an 8 word telegram from London to Pittsburg cost, at 25¢ a word to New York, and 25¢ for a message of less than 10 words from New York to Pittsburg?

35. A man bought 153 tons of coal at $4.25 a ton, and paid for one third of it at the time of purchase. He sold 36 cords of wood at $5.75 a cord and made another payment with the money. How much did he then owe?

36. Find the cost of 1728 doz. buttons at 58¢ a gross.

How many:

37. Yards equal 92 feet?

38. Dozen equal 32 gross?

39. Pints equal 37 gallons?

40. Pecks equal 26 bushels?

41. Pounds equal 128 oz.?

42. Quarts equal 32 gallons?

43. Hours equal 2 weeks?

44. Rods equal 594 feet?

45. Seconds equal 1 day?

46. Yards equal 129 feet?

47. Pecks equal 328 quarts?

48. Pounds equal 2 tons?

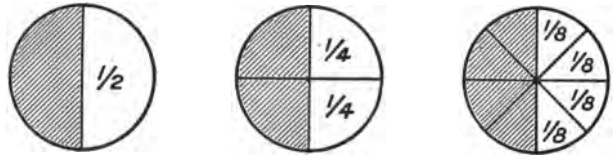
49. Ounces equal 450 lb.?

50. Feet equal 369 inches?

FRACTIONS

Fractional equivalents of halves, fourths, and eighths.

1 unit = $\frac{2}{2}$. 1 unit = $\frac{4}{4}$. 1 unit = $\frac{8}{8}$.



$\frac{1}{2}$ = $\frac{2}{4}$ = $\frac{4}{8}$

1. The first circle is divided into how many equal parts? What is each part called?

2. The second circle is divided into how many equal parts? What is each part called?

3. The third circle is divided into how many equal parts? What is each part called?

4. $\frac{1}{2}$ of the first circle = $\frac{?}{4}$ of the second circle. $\frac{1}{2}$ of the first circle = $\frac{?}{8}$ of the third circle.

5. How many halves of a circle are there in one circle? how many fourths of a circle? how many eighths of a circle?

6. Change $\frac{1}{2}$ to fourths; thus, $\frac{1}{2} = \frac{2}{4}$.

7. Change $\frac{1}{2}$ to eighths; $\frac{2}{2}$ to eighths.

8. How many units are there in $\frac{2}{2}$, $\frac{4}{4}$, $\frac{8}{8}$?

9. $\frac{1}{2}$ of the first circle + $\frac{1}{2}$ of the first circle = how many times the first circle? Then $\frac{1}{2} + \frac{1}{2}$ = how many?

10. $\frac{1}{4}$ of the second circle + $\frac{1}{4}$ of the second circle = what part of the second circle? Then $\frac{1}{4} + \frac{1}{4}$ = how many?

11. $\frac{2}{4}$ of the second circle + $\frac{2}{4}$ of the second circle = how many times the second circle? Then $\frac{2}{4} + \frac{2}{4}$ = how many?

12. $\frac{4}{8}$ of the third circle = $\frac{?}{2}$ of the same circle. Then $\frac{4}{8} = \frac{?}{2} = \frac{?}{4}$.

13. $\frac{1}{4} = \frac{?}{8}$; $\frac{1}{2} = \frac{?}{8} = \frac{?}{4}$.

14. Change $\frac{1}{2}$ and $\frac{1}{4}$ each to eighths.

15. $\frac{1}{2}$ of an orange = $\frac{?}{4}$ = $\frac{?}{8}$ of the same orange.

16. Four boys each have $\$\frac{1}{2}$. How many dollars have they?

17. $\$\frac{1}{4} + \$\frac{1}{4} = \$\frac{?}{4}$; $\$\frac{1}{2} + \$\frac{1}{4} = \$\frac{?}{4}$; $\frac{1}{8}$ day + $\frac{1}{2}$ day = $\frac{?}{8}$ day.

18. $\$\frac{4}{4} + \$\frac{4}{4} = \$\frac{?}{4}$; $\$\frac{8}{4}$ = how many dollars?

19. $\frac{8}{4}$ days = —days; $\frac{6}{4}$ days = 1 day and $\frac{?}{4}$ days.

20. $\$\frac{1}{4} + \$\frac{1}{2} = \$\frac{?}{4}$; $\$\frac{1}{2} + \$\frac{1}{2} + \$\frac{1}{2} + \$\frac{1}{2} = \$\frac{?}{2}$.

21. Write in order of their size $\frac{1}{2}$ of the first circle; $\frac{3}{4}$ of the second circle, and $\frac{3}{8}$ of the third circle.

A **unit** is any one thing.

A **fraction** is one or more of the equal parts of a unit.

22. Write in figures one half; one fourth. How many figures are needed to express a common fraction? In the fraction $\frac{3}{4}$ what does the 4 show? the 3?

23. Read $\frac{1}{2}$; $\frac{1}{3}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{4}{5}$; $\frac{5}{6}$; $\frac{7}{12}$.

24. Write seven eighths; thirteen twenty-fourths.

The **denominator** of a fraction, which is written below the line, shows into how many equal parts the unit is divided.

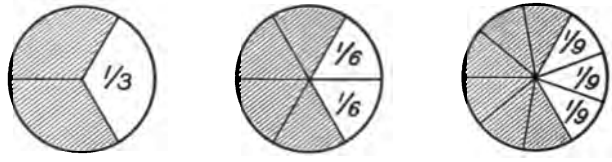
The **numerator** of a fraction, which is written above the line, shows how many equal parts of the fraction are taken.

The **terms** of a fraction are the numerator and denominator.

Fractions are said to be **equivalent** when they have the same value. Thus $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$ are equivalent fractions.

Fractional equivalents of thirds, sixths, and ninths.

1 unit = $\frac{3}{3}$. 1 unit = $\frac{6}{6}$. 1 unit = $\frac{9}{9}$.



$\frac{1}{3}$ = $\frac{2}{6}$ = $\frac{3}{9}$

1. Into how many parts is the first circle divided? the second circle? the third circle?

2. $\frac{1}{3}$ of a circle = $\frac{?}{6}$ of the circle = $\frac{?}{9}$ of the circle.

3. $\frac{2}{3}$ of a circle = $\frac{?}{6}$ of the circle = $\frac{?}{9}$ of the circle.

4. $\frac{3}{3}$ of a circle = $\frac{?}{6}$ of the circle = $\frac{?}{9}$ of the circle.

5. $\frac{1}{3} + \frac{2}{6} = \frac{?}{6}$; $\frac{1}{3} + \frac{3}{9} = \frac{?}{9}$; $\frac{1}{3} + \frac{3}{6} = \frac{?}{6}$.

6. $\frac{1}{3}$ of an hour + $\frac{1}{6}$ of an hour = $\frac{?}{6}$ of an hour.

7. $\frac{2}{3}$ of a day + $\frac{3}{9}$ of a day = $\frac{?}{9}$ of a day.

8. $\frac{1}{3} + \frac{1}{6} =$ —; $\frac{1}{3} + \frac{1}{9} =$ —; $\frac{1}{2} + \frac{1}{6} =$ —; $\frac{1}{4} + \frac{1}{8} =$ —.

9. Add $\frac{1}{3} + \frac{1}{3} + \frac{1}{3}$; $\frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6}$; $\frac{1}{2} + \frac{1}{2}$.

10. Draw an oblong and show that $\frac{1}{3}$ of the oblong = $\frac{2}{6}$ of the oblong = $\frac{3}{9}$ of the oblong.

11. Change to equivalent fractions in eighths: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$.

12. $\frac{1}{3} = \frac{?}{6}$; $\frac{2}{3} = \frac{?}{6}$; $\frac{2}{3} = \frac{?}{9}$; $\frac{3}{4} = \frac{?}{8}$.

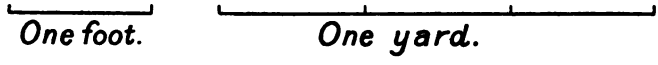
13. How many fourths of a pie = $\frac{6}{8}$ of a pie?

14. Draw squares and show that $\frac{1}{2} = \frac{4}{8}$; that $\frac{3}{4} = \frac{6}{8}$; that $\frac{1}{3} = \frac{3}{9}$; that $\frac{2}{3} = \frac{6}{9}$.

15. How many halves equal one unit? how many thirds? how many fourths? how many sixths? how many ninths?

16. $\frac{4}{2}$ = how many units? $\frac{12}{6}$ = how many units? $\frac{8}{4}$ = how many units? $\frac{3}{3}$ = how many units?

Fractional equivalents of the yard and the foot, and their sum.



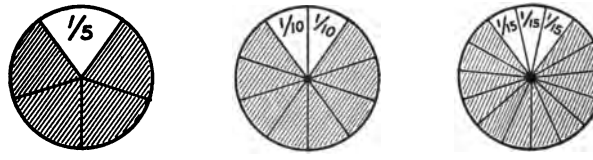
1. A foot is what part of a yard?
2. 2 feet are what part of a yard?
3. Into how many parts is the yard divided?
4. How many feet equal $\frac{1}{3}$ of a yard? $\frac{2}{3}$ of a yard? $\frac{3}{3}$ of a yard?
5. Measure a yard on the blackboard. Divide the yard into feet. Divide a foot into inches.
6. How many inches equal $\frac{1}{2}$ of a foot? $\frac{2}{3}$ of a foot? $\frac{3}{4}$ of a foot? $1\frac{1}{2}$ feet? $2\frac{1}{2}$ feet?
7. 6 inches are what part of a foot? of 2 feet? of a yard?
8. 4 inches are what part of a foot? of 2 feet?
9. $1\frac{1}{2}$ ft. $+ 1\frac{1}{2}$ ft. $=$ — ft.
10. $\frac{1}{2} = \frac{?}{4}$; $\frac{1}{2} + \frac{1}{4} = \frac{?}{4}$; $\frac{1}{2} = \frac{?}{6}$; $\frac{1}{2} + \frac{3}{6} = \frac{?}{6}$.
11. $1\frac{1}{4}$ ft. $+ \frac{1}{2}$ ft. $=$ how many ft.? $1\frac{1}{4} + \frac{1}{2} =$ —.
12. $\frac{1}{3}$ yd. $+ \frac{1}{2}$ yd. $+ \frac{1}{6}$ yd. $=$ — yd. $=$ — ft.
13. $2\frac{1}{3}$ ft. $+ 3\frac{2}{3}$ ft. $=$ — ft. $=$ — yd.
14. $\frac{3}{4}$ ft. $+ \frac{1}{4}$ ft. $=$ — ft.; $\frac{3}{4}$ ft. $+ \frac{3}{4}$ ft. $+ \frac{2}{4}$ ft. $=$ — ft.
15. $\frac{4}{2}$ ft. $=$ — ft.; $\frac{6}{3}$ ft. $=$ — ft.; $\frac{6}{4}$ ft. $=$ — ft.

Add:

16. $2\frac{3}{4}$ in. + $3\frac{1}{2}$ in.
17. $5\frac{1}{2}$ yd. + $8\frac{1}{6}$ yd.
18. $\frac{1}{2}$ ft. + $2\frac{1}{4}$ ft.
19. $2\frac{1}{3}$ ft. + $8\frac{5}{6}$ ft.
20. $3\frac{1}{4}$ ft. + $2\frac{3}{4}$ ft. + $3\frac{1}{2}$ ft.
21. $7\frac{1}{3}$ yd. + $6\frac{1}{6}$ yd. + $5\frac{1}{2}$ yd.
22. $\frac{1}{3}$ ft. + $2\frac{5}{6}$ ft. + $\frac{5}{6}$ ft.
23. $10\frac{1}{2}$ ft. + $15\frac{3}{4}$ ft. + $20\frac{1}{2}$ ft.

Fractional equivalents of fifths, tenths, and fifteenths, and their sum and difference.

1 unit = $\frac{5}{5}$. 1 unit = $\frac{10}{10}$. 1 unit = $\frac{15}{15}$.



$\frac{1}{5}$ = $\frac{2}{10}$ = $\frac{3}{15}$

1. Into how many parts is the first circle divided? the second circle? the third circle?

2. Observe the parts of each circle that are not shaded. $\frac{1}{5} = \frac{?}{10} = \frac{?}{15}$.

3. Then $\frac{2}{5} = \frac{?}{10} = \frac{?}{15}$; $\frac{3}{5} = \frac{?}{10} = \frac{?}{15}$; $\frac{4}{5} = \frac{?}{10} = \frac{?}{15}$.

4. Each of five boys had $\frac{1}{5}$ of a dollar. How many dollars did they have?

5. What is meant by $\frac{1}{5}$ of a circle? $\frac{2}{5}$ of a circle? $\frac{3}{5}$ of a circle? $\frac{1}{10}$ of a circle? $\frac{1}{15}$ of a circle?

6. $\frac{2}{5}$ of a circle + $\frac{2}{5}$ of the same circle = $\frac{?}{5}$ of the circle. Then $\frac{2}{5} + \frac{2}{5} = \frac{?}{5}$; $\frac{4}{5} - \frac{2}{5} = \frac{?}{5}$; $\frac{4}{5} - \frac{3}{5} = \frac{?}{5}$.

7. How many parts of a unit are there in $\frac{1}{5} + \frac{1}{5} + \frac{1}{5}$? in $\frac{2}{5} + \frac{2}{5}$? in $\frac{2}{10} + \frac{5}{10} + \frac{3}{10}$? in $\frac{3}{15} + \frac{8}{15} + \frac{3}{15}$? in $\frac{8}{15} - \frac{3}{15}$?

8. $\frac{1}{5} + \frac{1}{10} = \frac{?}{80}$; $\frac{1}{5} + \frac{1}{15} = \frac{?}{15}$; $\frac{1}{3} + \frac{1}{5} = \frac{?}{15}$.

9. $\frac{5}{5} + \frac{5}{5} = \frac{?}{5}$? Then $\frac{10}{5}$ = how many units?

10. $\frac{8}{5}$ = how many units and $\frac{3}{5}$ remaining?

11. $\frac{12}{5}$ = how many units and $\frac{2}{5}$ remaining?

12. $\frac{15}{10}$ = how many units and $\frac{5}{10}$ remaining?

13. Change to units and parts of units: $\frac{6}{5}$, $\frac{8}{5}$, $\frac{12}{5}$, $\frac{12}{10}$.

Add:

14. $\$2\frac{1}{5}$ / $3\frac{1}{10}$

15. $25\frac{3}{5}$ mi. / $4\frac{1}{5}$ mi.

16. $\$11\frac{1}{5}$ / $5\frac{3}{10}$

17. $5\frac{3}{16}$ / $4\frac{1}{8}$

18. $24\frac{1}{5}$ mi. / $4\frac{12}{15}$ mi. / $5\frac{1}{5}$ mi.

19. $\$23\frac{1}{5}$ / $7\frac{3}{10}$ / $31\frac{4}{5}$

20. $50\frac{1}{5}$ rd. / $35\frac{3}{15}$ rd. / $4\frac{4}{5}$ rd.

21. $24\frac{1}{4}$ da. / $3\frac{7}{8}$ da. / $4\frac{1}{2}$ da.

Subtract:

22. $\$3\frac{1}{5}$ / $2\frac{1}{10}$

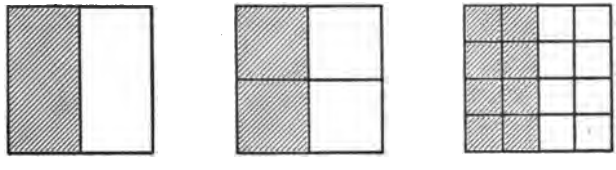
23. $25\frac{1}{5}$ yd. / $13\frac{1}{15}$ yd.

24. $14\frac{2}{5}$ bu. / $10\frac{1}{10}$ bu.

25. $78\frac{3}{8}$ gal. / $42\frac{1}{16}$ gal.

Fractional equivalents of halves, fourths, and sixteenths, and their sum and difference.

1 unit = $\frac{2}{2}$. 1 unit = $\frac{4}{4}$. 1 unit = $\frac{16}{16}$.



$\frac{1}{2}$ = $\frac{2}{4}$ = $\frac{8}{16}$

1. How do these three units compare in size?

2. Into how many parts is the first square divided? the second square? the third square?

3. $\frac{1}{2}$ of the first square = — fourths of the second square = — sixteenths of the third square.

4. $\frac{2}{2}$ = —units; $\frac{4}{4}$ = —units; $\frac{8}{8}$ = —units; $\frac{16}{16}$ = — units.

5. $\frac{1}{2} = \frac{?}{4} = \frac{?}{8} = \frac{?}{16}$.

6. $\frac{1}{4} = \frac{?}{8} = \frac{?}{16}$; $\frac{1}{8} = \frac{?}{16}$.

7. $\frac{2}{4} = \frac{?}{8} = \frac{?}{16}$; $\frac{8}{4} = \frac{?}{8} = \frac{?}{16}$.

8. $\frac{2}{8} = \frac{?}{16} = \frac{?}{4}$; $\frac{4}{8} = \frac{?}{16} = \frac{?}{2}$; $\frac{6}{8} = \frac{?}{16} = \frac{?}{4}$.

9. $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{?}{8}$; $\frac{1}{4} + \frac{3}{8} + \frac{3}{16} = \frac{?}{16}$; $\frac{9}{16} - \frac{1}{4} = \frac{?}{16}$.

Add:

10. $3\frac{1}{8}$ ft., $5\frac{1}{8}$ ft., $2\frac{1}{2}$ ft., $3\frac{1}{2}$ ft.

11. $16\frac{1}{2}$ ft., $20\frac{1}{4}$ ft., $17\frac{1}{8}$ ft., $10\frac{3}{16}$ ft.

12. $12\frac{1}{4}$, $14\frac{1}{2}$, $10\frac{5}{8}$, $12\frac{1}{2}$

13. $10\frac{3}{8}$, $5\frac{1}{2}$, $6\frac{3}{4}$, $8\frac{3}{16}$

Subtract:

14. \$ $12\frac{1}{2}$ − $8\frac{1}{4}$

15. $23\frac{3}{4}$ yd. − $18\frac{1}{16}$ yd.

16. $13\frac{1}{2}$ mi. − $9\frac{3}{16}$ mi.

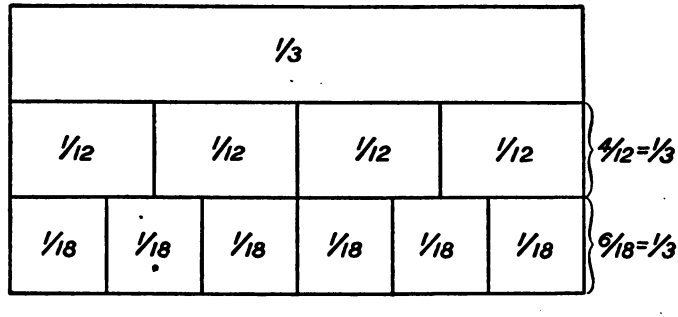
17. $68\frac{3}{4}$ − $52\frac{3}{16}$

18. A flower bed is 4 ft. 6 in. long and 3 ft. 4 in. wide. Find the distance around it.

19. The school ground is in the form of a square, $13\frac{1}{2}$ rd. on a side. Find the distance in rods around it.

Fractional equivalents of sixths, twelfths, and eighteenths, and their sum and difference.

1 unit $= \frac{3}{3} = \frac{12}{12} = \frac{18}{18}$.



1. Into how many thirds can the oblong be divided? into how many twelfths? into how many eighteenths?

2. $\frac{1}{3} = \frac{?}{12} = \frac{?}{18}$; $\frac{2}{3} = \frac{?}{12} = \frac{?}{18}$.

3. $\frac{2}{3}$ of a day = how many 9ths of a day? how many 18ths of a day?

4. $\frac{1}{2}$ hour = $\frac{?}{12}$ of an hour; = $\frac{?}{18}$ of an hour.

5. Change to 18ths: $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{9}$.

6. Change to 16ths: $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$.

7. $\frac{6}{3}$ = how many units? $\frac{12}{3}$ = how many units?

8. Change to 3ds: $\frac{6}{18}$, $\frac{6}{9}$, $\frac{12}{18}$, $\frac{3}{9}$.

9. Draw oblongs and show that $\frac{1}{3} = \frac{2}{6}$; $\frac{4}{8} = \frac{1}{2}$; $\frac{9}{18} = \frac{1}{2}$; $\frac{12}{18} = \frac{2}{3}$; $\frac{9}{12} = \frac{3}{4}$.

10. $\frac{18}{18}$ = how many units? $\frac{20}{18}$ = how many units and $\frac{2}{18}$ remaining?

11. Change to units and parts of units: $\frac{3}{2}$, $\frac{9}{6}$, $\frac{10}{8}$, $\frac{12}{6}$, $\frac{15}{10}$, $\frac{8}{5}$, $\frac{16}{10}$, $\frac{16}{15}$, $\frac{20}{16}$.

Add:

12.
$29\frac{1}{4}$
$32\frac{1}{6}$
$45\frac{5}{12}$

13.
$7\frac{1}{16}$
$10\frac{3}{4}$
$25\frac{7}{8}$

14.
$39\frac{1}{6}$
$42\frac{5}{12}$
$23\frac{7}{18}$

15.
$5\frac{1}{3}$
$6\frac{5}{6}$
$12\frac{7}{18}$

16.
$27\frac{3}{4}$ ft.
$45\frac{7}{8}$ ft.
$25\frac{3}{16}$ ft.

17.
$15\frac{1}{6}$ mi.
$29\frac{4}{15}$ mi.
$31\frac{1}{3}$ mi.

18.
$14\frac{1}{8}$ bu.
$19\frac{1}{4}$ bu.
$16\frac{5}{8}$ bu.

19.
$12\frac{1}{3}$ da.
$10\frac{1}{4}$ da.
$13\frac{1}{12}$ da.

Subtract:

20.
$8\frac{1}{3}$
$6\frac{1}{12}$

21.
$17\frac{1}{3}$
$15\frac{1}{18}$

22.
$14\frac{2}{3}$
$12\frac{1}{12}$

23.
$32\frac{5}{12}$
$30\frac{1}{3}$

24.
$7\frac{2}{3}$ in.
$5\frac{1}{12}$ in.

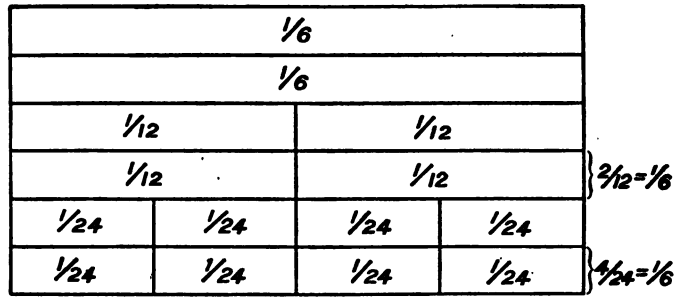
25.
$9\frac{2}{3}$ bu.
$7\frac{1}{18}$ bu.

26.
$10\frac{1}{3}$ lb.
$6\frac{1}{21}$ lb.

27.
$15\frac{11}{12}$ da.
$5\frac{2}{3}$ da.

Fractional equivalents of sixths, twelfths, and twenty-fourths, and their sum and difference.

1 unit $= \frac{6}{6} = \frac{12}{12} = \frac{24}{24}$.



1. What part of the oblong $= \frac{3}{6}$ of it? $\frac{6}{12}$ of it? $\frac{12}{24}$ of it?
2. $\frac{5}{6}$ of the oblong $= \frac{?}{12}$ of the oblong; equals $\frac{?}{24}$ of it.
3. $\frac{4}{24}$ of the oblong $= \frac{?}{6}$ of the oblong; equals $\frac{?}{12}$ of it.
4. $\frac{3}{6} + \frac{3}{6} + \frac{3}{6} + \frac{3}{6} = \frac{?}{6}$; equals how many units?
5. Any unit can be divided into how many halves? 3ds? 4ths? 5ths? 6ths? 7ths? 8ths? 16ths? 24ths, etc.?
6. Add $\frac{1}{6}$ and $\frac{1}{12}$; $\frac{1}{12}$ and $\frac{1}{24}$. From $\frac{5}{24}$ take $\frac{1}{12}$.
7. From $\frac{23}{24}$ subtract $\frac{1}{6}$; $\frac{5}{12}$; $\frac{5}{6}$; $\frac{7}{12}$.
8. $\frac{8}{5}$ means that a unit ($\frac{5}{5}$) and a part of a unit ($\frac{3}{5}$) have been added. What does $\frac{8}{7}$ mean? $\frac{9}{8}$?

Add:

9.	10.	11.	12.
$18\frac{1}{6}$ in.	$15\frac{5}{6}$ bu.	$19\frac{1}{6}$	$40\frac{7}{24}$
$20\frac{1}{12}$ in.	$27\frac{1}{24}$ bu.	$32\frac{5}{12}$	$30\frac{1}{12}$
$39\frac{1}{24}$ in.	$41\frac{10}{12}$ bu.	$20\frac{5}{24}$	$18\frac{1}{6}$

Subtract:

13.	14.	15.	16.
$9\frac{1}{6}$ yd.	$14\frac{5}{6}$ da.	$28\frac{7}{12}$	$39\frac{5}{6}$
$7\frac{1}{12}$ yd.	$7\frac{5}{12}$ da.	$13\frac{5}{24}$	$8\frac{7}{24}$

REDUCTION OF FRACTIONS

1. Notice in the diagram on p. 30 that $\frac{1}{6} = \frac{4}{24}$. By what number are both numerator and denominator of $\frac{1}{6}$ *multiplied* to change it to $\frac{4}{24}$? Is there any difference in *value* between $\frac{1}{6}$ and $\frac{4}{24}$? Notice that the terms in $\frac{4}{24}$ are larger or *higher* than in $\frac{1}{6}$. The change of $\frac{1}{6}$ to the equal fraction $\frac{4}{24}$ is called **changing or reducing $\frac{1}{6}$ to higher terms.**

2. By what number must both terms of $\frac{4}{24}$ be *divided* to change $\frac{4}{24}$ to $\frac{1}{6}$? Is there any difference in *value* between $\frac{4}{24}$ and $\frac{1}{6}$? Which fraction has the **lower terms**? The change of $\frac{4}{24}$ to $\frac{1}{6}$ is called **reducing $\frac{4}{24}$ to lower terms.**

3. Notice in the diagram that $\frac{4}{24} = \frac{2}{12} = \frac{1}{6}$. When $\frac{4}{24}$ is changed to $\frac{2}{12}$ it is reduced to *lower* terms but not to its *lowest* terms, since $\frac{2}{12}$ can be changed to still lower terms, $\frac{1}{6}$. Can $\frac{1}{6}$ be reduced to still lower terms? The change of $\frac{4}{24}$ to $\frac{1}{6}$ is called **reducing $\frac{4}{24}$ to its lowest terms.**

4. By what number must both terms of $\frac{1}{2}$ be multiplied to change it to the equal fraction $\frac{3}{6}$? By what number must both terms of $\frac{8}{10}$ be divided to change it to the equal fraction $\frac{4}{5}$? Is $\frac{4}{5}$ in its lowest terms?

Multiplying or dividing both terms of a fraction by the same number does not alter its value.

5. Reduce to higher terms: $\frac{1}{2}$; $\frac{2}{3}$; $\frac{3}{4}$; $\frac{4}{5}$; $\frac{5}{6}$; $\frac{7}{8}$; $\frac{2}{9}$; $\frac{1}{10}$.

6. Reduce to lowest terms: $\frac{2}{4}$; $\frac{4}{8}$; $\frac{4}{6}$; $\frac{3}{9}$; $\frac{4}{16}$; $\frac{8}{10}$.

Fractions like $\frac{1}{5}$, $\frac{2}{5}$, and $\frac{4}{5}$, which have the same denominator, are said to have a **common denominator.**

Similar fractions are fractions that have a common denominator.

7. Change to similar fractions $\frac{1}{2}$ and $\frac{2}{4}$; $\frac{1}{3}$ and $\frac{1}{6}$; $\frac{1}{6}$ and $\frac{1}{12}$; $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{3}{8}$.

Written Work

1. Change $\frac{2}{3}$ and $\frac{3}{4}$ to similar fractions having the *common denominator* 12.

$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$ Since the denominator 3 in $\frac{2}{3}$ must be multiplied by 4 to produce 12, the numerator must also be multiplied by 4, so as not to change the value of the fraction. $\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$.

$\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$ Since the denominator 4 in $\frac{3}{4}$ must be multiplied by 3 to produce 12, the numerator 3 must also be multiplied by 3. $\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$. Hence the similar fractions are $\frac{8}{12}$ and $\frac{9}{12}$.

Change to similar fractions:

2. $\frac{1}{2}$ and $\frac{2}{5}$
3. $\frac{3}{4}$ and $\frac{5}{6}$
4. $\frac{2}{3}$ and $\frac{5}{8}$
5. $\frac{4}{9}$ and $\frac{1}{4}$
6. $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{3}{8}$
7. $\frac{2}{3}$, $\frac{1}{2}$, and $\frac{5}{6}$
8. $\frac{1}{3}$, $\frac{2}{9}$, and $\frac{4}{18}$
9. $\frac{5}{6}$, $\frac{3}{8}$, and $\frac{1}{24}$

10. Reduce $\frac{6}{24}$ to its lowest terms.

$\frac{6 \div 3}{24 \div 3} = \frac{2}{8}$; $\frac{2 \div 2}{8 \div 2} = \frac{1}{4}$ We can divide both terms, 6 and 24, of the fraction $\frac{6}{24}$ by 3 without changing the *value* of the fraction. The result is $\frac{2}{8}$. We can then divide both terms 2 and 8 of the fraction $\frac{2}{8}$ by 2. The result is $\frac{1}{4}$.

Reduce to lowest terms:

11. $\frac{8}{24}$
12. $\frac{9}{27}$
13. $\frac{10}{25}$
14. $\frac{8}{32}$
15. $\frac{18}{36}$
16. $\frac{14}{21}$
17. $\frac{16}{40}$
18. $\frac{14}{35}$
19. $\frac{12}{36}$
20. $\frac{15}{35}$

21. Change $\frac{4}{2}$ to units. Thus, 1 unit = 2 halves. In 4 halves there are $4 \div 2$, or 2, units. Change $\frac{10}{5}$ to units.

22. Change $\frac{10}{3}$ to units and parts of units. Thus, 1 = 3 thirds. In 10 thirds there are $10 \div 3$, or 3, units and $\frac{1}{3}$ remaining; that is, $3\frac{1}{3}$.

To change a fraction to units and parts of units, divide the numerator by the denominator.

Change to units and parts of units:

23. $\frac{12}{6}$
24. $\frac{12}{8}$
25. $\frac{10}{6}$
26. $\frac{12}{8}$
27. $\frac{17}{4}$

ADDITION OF FRACTIONS

1. Can you add $\frac{1}{8}$ and $\frac{1}{6}$ without change? Can you add $\frac{2}{6}$ and $\frac{1}{6}$? What change must be made in $\frac{2}{3}$ and $\frac{1}{4}$ before they can be added?

2. $\frac{1}{5} = \frac{?}{10}$; $\frac{2}{5} = \frac{?}{10}$; $\frac{3}{5} = \frac{?}{10}$; $\frac{5}{5} = \frac{?}{10}$?

3. $\frac{1}{2} = \frac{?}{10}$; $\frac{1}{2} + \frac{1}{5} = \frac{?}{10}$; $\frac{1}{2} + \frac{1}{3} = \frac{?}{6}$; $\frac{1}{2} + \frac{1}{8} = \frac{?}{8}$?

4. Can you add $\frac{1}{2}$ and $\frac{1}{5}$ without change? Change both to tenths. Can they then be added?

5. Can you add $\frac{1}{2}$ and $\frac{1}{3}$ without change? Change both to sixths. Can they then be added?

6. When $\frac{1}{2}$ and $\frac{1}{4}$ are to be added, to what similar fractions should they be changed?

7. What are the denominators of the fractions in example 4? To what like or *common denominators* (c. d.) did you change both fractions?

8. What are the denominators of the fractions in example 5? To what denominator did you change the fraction $\frac{1}{2}$? $\frac{1}{3}$? Why?

9. After two or more fractions are changed to like, or *common denominators*, that is, after they have been made *similar*, what is the *second step in adding them?*

10. Add $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$; $\frac{1}{4}$, $\frac{1}{8}$; $\frac{1}{5}$, $\frac{1}{10}$; $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{12}$; $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{15}$.

11. Observe that in problem 10, $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{6}{6}$, or 1, and that $\frac{1}{3} + \frac{1}{4} + \frac{1}{12} = \frac{8}{12}$, or $\frac{2}{3}$.

12. What is the *third* step in adding fractions?

Why is the first step not necessary in the following?

13. $\frac{2}{3} + \frac{1}{3}$ 15. $\frac{1}{5} + \frac{3}{5}$ 17. $\frac{1}{8} + \frac{3}{8} + \frac{5}{8} + \frac{7}{8}$

14. $\frac{4}{9} + \frac{5}{9}$ 16. $\frac{3}{7} + \frac{4}{7}$ 18. $\frac{1}{10} + \frac{3}{10} + \frac{5}{10} + \frac{9}{10}$

Give the sums at sight:

19. $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$
20. $\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$
21. $\frac{1}{5}+\frac{1}{5}+\frac{1}{5}$
22. $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$
23. $\frac{1}{8}+\frac{2}{8}+\frac{1}{8}+\frac{3}{8}$
24. $\frac{1}{7}+\frac{2}{7}+\frac{2}{7}+\frac{1}{7}$
25. $\frac{1}{10}+\frac{2}{10}+\frac{2}{10}+\frac{3}{10}$
26. $\frac{1}{12}+\frac{4}{12}+\frac{3}{12}$
27. $\frac{1}{9}+\frac{2}{9}+\frac{3}{9}+\frac{2}{9}$
28. $\frac{1}{14}+\frac{3}{14}+\frac{5}{14}+\frac{2}{14}$
29. $\frac{2}{15}+\frac{3}{15}+\frac{5}{15}+\frac{4}{15}$
30. $\frac{1}{11}+\frac{2}{11}+\frac{5}{11}+\frac{2}{11}$
31. $\frac{1}{16}+\frac{3}{16}+\frac{5}{16}+\frac{2}{16}$
32. $\frac{1}{20}+\frac{4}{20}+\frac{6}{20}+\frac{8}{20}$
33. $\frac{3}{13}+\frac{4}{13}+\frac{1}{13}+\frac{2}{13}$
34. $\frac{5}{25}+\frac{8}{25}+\frac{2}{25}+\frac{4}{25}$
35. $\frac{7}{18}+\frac{2}{18}+\frac{7}{18}+\frac{1}{18}$
36. $\frac{3}{17}+\frac{5}{17}+\frac{4}{17}+\frac{5}{17}$

37. A boy spent $\frac{1}{4}$ of his money for a knife, $\frac{1}{4}$ of it for a ball, and $\frac{1}{4}$ of it for his lunch. What part of his money did he spend?

38. A grocer sold $\frac{1}{8}$ of a pound of pepper to one customer, $\frac{2}{8}$ of a pound to another, and $\frac{4}{8}$ of a pound to another. What part of a pound did he sell?

39. I paid $\$\frac{1}{10}$ for milk, $\$\frac{2}{10}$ for lettuce, and $\$\frac{8}{10}$ for butter. What part of a dollar did I pay for all?

40. David paid $\frac{3}{5}$ of a dollar for a fishing rod, and $\frac{1}{5}$ of a dollar for a line. How much did he pay for both?

Adding fractions that are not similar.

Written Work

1. Add $\frac{2}{3}$ and $\frac{1}{4}$.

$12 = \text{c. d.}$

$\frac{2 \times 4}{3 \times 4} = \frac{8}{12}$

$\frac{1 \times 3}{4 \times 3} = \frac{3}{12}$

$\frac{2}{3}+\frac{1}{4}=\frac{11}{12}$

The fractions must first be made similar. They may be changed to the common denominator twelfths. Multiplying both terms of $\frac{2}{3}$ by 4 changes it to $\frac{8}{12}$, and multiplying both terms of $\frac{1}{4}$ by 3 changes it to $\frac{3}{12}$. The sum of $\frac{8}{12}$ and $\frac{3}{12}$ is $\frac{11}{12}$.

Fractions must be made similar before they can be added.

Add, using a pencil; then orally:

2. $\frac{1}{2}$ and $\frac{1}{3}$	8. $\frac{1}{4}$ and $\frac{1}{6}$	14. $\frac{1}{3}$ and $\frac{1}{6}$
3. $\frac{1}{2}$ and $\frac{1}{6}$	9. $\frac{1}{2}$ and $\frac{1}{8}$	15. $\frac{1}{5}$ and $\frac{1}{6}$
4. $\frac{1}{3}$ and $\frac{1}{4}$	10. $\frac{1}{4}$ and $\frac{1}{5}$	16. $\frac{1}{3}$ and $\frac{1}{12}$
5. $\frac{1}{2}$ and $\frac{1}{4}$	11. $\frac{1}{3}$ and $\frac{1}{5}$	17. $\frac{1}{2}$ and $\frac{1}{5}$
6. $\frac{1}{3}$ and $\frac{1}{7}$	12. $\frac{1}{7}$ and $\frac{1}{4}$	18. $\frac{1}{3}$ and $\frac{1}{9}$
7. $\frac{1}{2}$ and $\frac{1}{7}$	13. $\frac{1}{2}$ and $\frac{1}{8}$	19. $\frac{1}{3}$ and $\frac{1}{12}$

20. Henry had $\frac{1}{5}$ of a dollar, and found $\frac{1}{4}$ of a dollar. How much had he then?

21. Mary bought $\frac{1}{3}$ of a yard of red ribbon, $\frac{1}{4}$ of a yard of blue ribbon, and $\frac{1}{2}$ of a yard of white ribbon. How many yards of ribbon did she buy?

22. What is the total cost of a ball at $\frac{1}{2}$ of a dollar, a penknife at $\frac{1}{5}$ of a dollar, and a book at $\frac{1}{4}$ of a dollar?

A **mixed number** is a number expressed by a whole number and a fraction, as $5\frac{1}{4}$, $3\frac{1}{3}$, $17\frac{1}{2}$.

Adding mixed numbers when the sum of the fractions is less than a whole unit.

Written Work

1. Add $2\frac{1}{3}$ and $3\frac{1}{4}$.

$12 = \text{c. d.}$

$2\frac{1}{3} = 2\frac{4}{12}$

$3\frac{1}{4} = 3\frac{3}{12}$

$2\frac{1}{3} + 3\frac{1}{4} = 5\frac{7}{12}$

$\frac{1}{3}$ and $\frac{1}{4}$ may each be changed to twelfths. Write the common denominator (c. d.), 12, above the fractions. $\frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$; $\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$. The sum of the fractions is $\frac{7}{12}$ and the sum of the integers is 5; $5 + \frac{7}{12} = 5\frac{7}{12}$.

Add:

2.	3.	4.	5.
$5\frac{1}{2}$	$12\frac{1}{3}$	$1\frac{1}{2}$	$85\frac{1}{4}$
$6\frac{1}{3}$	$14\frac{1}{4}$	$20\frac{1}{6}$	$60\frac{1}{5}$
1	2	$8\frac{1}{12}$	$42\frac{1}{20}$

Add:

6.	7.	8.	9.
$\frac{1}{2}$	$20\frac{1}{3}$	$111\frac{1}{5}$	$20\frac{1}{4}$
$40\frac{1}{5}$	$24\frac{1}{9}$	$\frac{1}{6}$	$145\frac{1}{16}$
$18\frac{1}{10}$	$28\frac{1}{18}$	$37\frac{1}{30}$	$230\frac{1}{8}$

10.	11.	12.	13.
$100\frac{1}{3}$	$35\frac{1}{5}$	$12\frac{1}{3}$	$128\frac{1}{2}$
$80\frac{1}{12}$	$6\frac{1}{8}$	$10\frac{1}{6}$	$3\frac{1}{5}$
$205\frac{1}{36}$	$117\frac{1}{40}$	$1\frac{1}{9}$	$9\frac{1}{20}$

14. A man walked $4\frac{1}{3}$ miles one hour, $4\frac{1}{4}$ miles the second hour, and $3\frac{1}{4}$ miles the third hour. How far did he walk?

15. A farmer sold corn for $\$14\frac{1}{4}$, wheat for $\$37\frac{1}{5}$, and rye for $\$15\frac{1}{20}$. How much did he receive for all?

Adding mixed numbers when the sum of the fractions is greater than a whole unit.

Written Work

1. Add $8\frac{2}{3}$ and $12\frac{3}{5}$.

15 = c. d.

$8\frac{2}{3} = 8\frac{10}{15}$

$12\frac{3}{5} = 12\frac{9}{15}$

$8\frac{2}{3} + 12\frac{3}{5} = 20\frac{19}{15}$ or $21\frac{4}{15}$

$\frac{2}{3}$ and $\frac{3}{5}$ may each be changed to fifteenths. $\frac{2}{3} \times \frac{5}{5} = \frac{10}{15}$; $\frac{3}{5} \times \frac{3}{3} = \frac{9}{15}$. The sum of $\frac{19}{15}$ and $\frac{9}{15}$ is $\frac{19}{15}$, which equals $1\frac{4}{15}$. The 1 is added to the sum of 12 and 8, making 21, which with $\frac{4}{15}$, makes $21\frac{4}{15}$.

Add:

2.	3.	4.	5.
$7\frac{2}{3}$	$150\frac{3}{4}$	$80\frac{5}{6}$	$32\frac{1}{2}$
$8\frac{1}{4}$	$68\frac{5}{8}$	$18\frac{3}{8}$	$60\frac{5}{12}$

6.	7.	8.	9.
$175\frac{1}{5}$	$350\frac{7}{12}$	$6\frac{5}{9}$	$17\frac{3}{4}$
$\frac{3}{10}$	$267\frac{3}{4}$	$14\frac{7}{18}$	$2\frac{1}{5}$
$16\frac{1}{2}$	$419\frac{2}{3}$	$22\frac{1}{3}$	$30\frac{9}{20}$

Add:

10.	11.	12.	13.
$80\frac{1}{4}$	$17\frac{7}{8}$	$20\frac{3}{5}$	$135\frac{1}{6}$
$18\frac{1}{6}$	$71\frac{3}{4}$	$120\frac{3}{4}$	$122\frac{4}{15}$
$42\frac{3}{8}$	$9\frac{9}{16}$	$261\frac{3}{10}$	$118\frac{7}{10}$
$12\frac{5}{24}$	$10\frac{1}{2}$	$268\frac{11}{20}$	$94\frac{11}{30}$

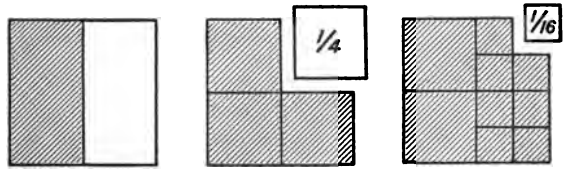
14. Mary bought a reader for $\frac{2}{5}$ of a dollar, a slate for $\frac{3}{10}$ of a dollar, and a grammar for $\frac{1}{2}$ of a dollar. How much did they all cost?

15. A clerk spent $\$18\frac{1}{2}$ a month for board, $\$9\frac{1}{5}$ for a room, and $\$4\frac{7}{20}$ for clothes. How much did he spend in one month?

16. I sold $\frac{1}{4}$ of an acre of land to one man, $2\frac{2}{3}$ acres to another, and $1\frac{5}{6}$ acres to another. How many acres did I sell?

17. Find the perimeter of a sheet of paper $9\frac{1}{4}$ in. by $5\frac{1}{8}$ in.

SUBTRACTION OF FRACTIONS



1. 1 sq. in. $-$ $\frac{1}{2}$ sq. in $=$ — sq. in.

2. $\frac{1}{2}$ sq. in. $-$ $\frac{1}{4}$ sq. in. $=$ — sq. in.

3. 1 sq. in. $-$ $\frac{1}{4}$ sq. in $=$ — sq. in.

4. $\frac{1}{2}$ sq. in. $-$ $\frac{4}{16}$ sq. in. $=$ — sq. in.

5. $\frac{1}{2}$ sq. in. $-$ $\frac{1}{16}$ sq. in. $=$ — sq. in.

6. $\frac{3}{4} - \frac{1}{4} = \frac{?}{4}$; $\frac{5}{8} - \frac{1}{8} = \frac{?}{8}$; $\frac{9}{10} - \frac{3}{10} = \frac{?}{10}$.

7. Give answers to the following: $\frac{11}{12} - \frac{5}{12}$; $\frac{11}{18} - \frac{5}{18}$; $\frac{6}{11} - \frac{4}{11}$; $\frac{19}{20} - \frac{3}{20}$; $\frac{24}{25} - \frac{3}{25}$.

8. What do you notice about the denominators of the fractions you have subtracted in example 7?

In subtraction of fractions, just as in subtraction of whole numbers, the minuend must be *larger* than the subtrahend.

9. When the denominators are alike, what do we subtract?

10. Could you subtract $\frac{1}{3}$ from $\frac{1}{2}$ without change? How may these fractions be made similar?

11. When the denominators are *unlike*, what is the *first step?* What is the *second step?* What is the *third step?*

Subtract:

12. \$$\frac{1}{2}$ − \$$\frac{1}{4}$; $\frac{2}{3}$ ft. − $\frac{1}{4}$ ft.; $\frac{7}{9}$ yd. − $\frac{2}{3}$ yd.; $\frac{11}{12}$ ft. − $\frac{7}{8}$ ft.

13. $\frac{9}{10} - \frac{2}{3}$; $\frac{16}{21} - \frac{3}{7}$; $\frac{11}{12} - \frac{2}{3}$; $\frac{4}{5} - \frac{1}{2}$; $\frac{27}{35} - \frac{2}{5}$; $\frac{7}{8} - \frac{1}{4}$.

Subtracting similar mixed numbers.

Written Work

1. From $3\frac{1}{3}$ take $1\frac{2}{3}$.

$3\frac{1}{3} = 2\frac{4}{3}$

$1\frac{2}{3} = 1\frac{2}{3}$

$3\frac{1}{3} - 1\frac{2}{3} = 1\frac{2}{3}$

Since $\frac{2}{3}$ cannot be subtracted from $\frac{1}{3}$, we take 1 or $\frac{3}{3}$ from 3, and add it to $\frac{1}{3}$, thus changing $3\frac{1}{3}$ to $2\frac{4}{3}$. Then $\frac{2}{3}$ from $\frac{4}{3}$ leaves $\frac{2}{3}$ and 1 from 2 leaves 1. Hence the answer is $1\frac{2}{3}$.

Find the differences:

2. $2\frac{5}{6} - 1\frac{4}{6}$

3. $5\frac{7}{8} - 2\frac{5}{8}$

4. $9\frac{11}{12} - 6\frac{4}{12}$

5. $20\frac{14}{15} - 11\frac{11}{15}$

6. $16\frac{19}{21} - 11\frac{7}{21}$

7. $35\frac{9}{10} - 5\frac{2}{10}$

8. $17\frac{10}{11} - 13\frac{2}{11}$

9. $11\frac{7}{9} - 9\frac{3}{9}$

10. $7\frac{4}{5} - 2\frac{3}{5}$

11. $13\frac{5}{7} - 9\frac{2}{7}$

12. If a boy buys papers at $\frac{3}{5}$ of a cent each, and sells them at 1 cent each, how much does he gain on each paper?

13. I bought a bushel of potatoes for $\frac{8}{10}$ of a dollar, and a bushel of corn for $\frac{5}{10}$ of a dollar. How much more did I pay for the potatoes than for the corn?

14. William bought a hat for $\$2\frac{3}{4}$, and a pair of shoes for $\$2\frac{1}{4}$. How much more did he pay for the hat than for the shoes?

15. A dealer sold eggs for $\$10\frac{17}{20}$ which cost him $\$9\frac{6}{20}$. How much did he gain?

16. One farmer owns $124\frac{15}{16}$ acres of land, and another owns $111\frac{8}{16}$ acres. How many more acres does the one farmer own than the other?

Subtracting fractions or mixed numbers that are not similar.

Written Work

1. From $\frac{1}{4}$ take $\frac{1}{5}$.

$20 = \text{c. d.}$

$\frac{1}{4} - \frac{1}{5}$

$\frac{5}{20} - \frac{4}{20} = \frac{1}{20}$

Before they can be subtracted the fractions must be made similar. Change them both to twentieths. $\frac{1}{4} = \frac{5}{20}$ and $\frac{1}{5} = \frac{4}{20}$. Subtracting, $\frac{5}{20} - \frac{4}{20} = \frac{1}{20}$.

Fractions must be made similar before they can be subtracted.

Subtract, using a pencil; then orally:

2. $\frac{1}{2} - \frac{1}{3}$

3. $\frac{1}{4} - \frac{1}{5}$

4. $\frac{1}{3} - \frac{1}{4}$

5. $\frac{1}{2} - \frac{1}{8}$

6. $\frac{1}{3} - \frac{1}{7}$

7. $\frac{1}{8} - \frac{1}{12}$

8. $\frac{1}{2} - \frac{1}{9}$

9. $\frac{1}{3} - \frac{1}{5}$

10. $\frac{1}{6} - \frac{1}{7}$

11. $\frac{1}{6} - \frac{1}{8}$

12. $\frac{1}{3} - \frac{1}{9}$

13. $\frac{1}{4} - \frac{1}{7}$

14. $\frac{1}{5} - \frac{1}{9}$

15. $\frac{1}{2} - \frac{1}{7}$

16. $\frac{1}{3} - \frac{1}{12}$

17. $\frac{1}{4} - \frac{1}{8}$

18. $\frac{1}{5} - \frac{1}{6}$

19. $\frac{1}{4} - \frac{1}{6}$

20. $\frac{1}{5} - \frac{1}{10}$

21. $\frac{1}{5} - \frac{1}{8}$

22. $\frac{1}{6} - \frac{1}{4}$

23. From a piece of cloth containing $\frac{1}{2}$ of a yard, $\frac{1}{4}$ of a yard was sold. What part of a yard remained?

24. From a city lot containing $\frac{1}{8}$ of an acre, $\frac{1}{16}$ of an acre was sold. What part of an acre remained?

25. A man traveled $\frac{1}{4}$ of a mile the first hour, and $\frac{1}{5}$ of a mile the second hour. How much farther did he travel the first hour than the second?

26. From 7 take $\frac{2}{3}$.

$$7 = 6\tfrac{3}{3}$$
$$\tfrac{2}{3} = \tfrac{2}{3}$$
$$7 - \tfrac{2}{3} = 6\tfrac{1}{3}$$

Change 7 into $6\frac{3}{3}$. Subtracting $\frac{2}{3}$ from $\frac{3}{3}$ gives $\frac{1}{3}$, which added to 6 gives $6\frac{1}{3}$.

Subtract, using a pencil; then orally:

27.	$3 - \frac{1}{2}$	**34.**	$22 - \frac{11}{12}$	**41.**	$133 - \frac{8}{9}$
28.	$12 - \frac{2}{3}$	**35.**	$7 - \frac{1}{3}$	**42.**	$44 - \frac{2}{25}$
29.	$18 - \frac{7}{8}$	**36.**	$28 - \frac{11}{14}$	**43.**	$11 - \frac{3}{4}$
30.	$9 - \frac{5}{6}$	**37.**	$55 - \frac{13}{16}$	**44.**	$40 - \frac{3}{5}$
31.	$3 - \frac{4}{7}$	**38.**	$4 - \frac{19}{20}$	**45.**	$7 - \frac{8}{11}$
32.	$100 - \frac{9}{10}$	**39.**	$125 - \frac{1}{18}$	**46.**	$51 - \frac{7}{10}$
33.	$18 - \frac{14}{15}$	**40.**	$10 - \frac{4}{5}$	**47.**	$48 - \frac{7}{8}$

48. Albert had \$2 and spent $\$\frac{4}{5}$ for skates. How much money had he remaining?

49. A vessel contained 8 gallons of oil. After $\frac{7}{8}$ of a gallon had leaked out, how much remained?

50. A grocer who had bought 10 bushels of potatoes, sold $\frac{3}{4}$ of a bushel. How many bushels remained?

51. From $12\frac{3}{4}$ take $10\frac{1}{2}$.

$$4 = \text{c. d.}$$
$$12\tfrac{3}{4} = 12\tfrac{3}{4}$$
$$10\tfrac{1}{2} = 10\tfrac{2}{4}$$
$$12\tfrac{3}{4} - 10\tfrac{1}{2} = 2\tfrac{1}{4}$$

Change $\frac{1}{2}$ to fourths. $\frac{1}{2} = \frac{2}{4}$. $\frac{3}{4}$ from $\frac{4}{4} = \frac{1}{4}$, which added to 12 less 10, or 2, gives $2\frac{1}{4}$.

Find differences:

52. $4\frac{3}{4} - 3\frac{1}{2}$

53. $7\frac{2}{3} - 4\frac{1}{2}$

54. $10\frac{5}{6} - 3\frac{1}{3}$

55. $10\frac{7}{8} - 2\frac{3}{4}$

56. $12\frac{7}{9} - 5\frac{2}{3}$

57. $24\frac{7}{10} - 11\frac{2}{5}$

58. $31\frac{11}{12} - 18\frac{5}{6}$

59. $79\frac{3}{16} - 26\frac{5}{8}$

60. $97\frac{17}{21} - 35\frac{4}{7}$

61. $121\frac{3}{4} - 66\frac{2}{5}$

62. $80\frac{5}{8} - 14\frac{1}{2}$

63. $98\frac{8}{9} - 32\frac{1}{3}$

64. $45\frac{23}{30} - 30\frac{2}{3}$

65. $25\frac{4}{9} - 12\frac{1}{4}$

66. $100\frac{1}{3} - 52\frac{1}{7}$

67. $78\frac{2}{3} - 35\frac{3}{5}$

68. $50\frac{4}{5} - 40\frac{1}{3}$

69. $124\frac{5}{11} - 112\frac{1}{3}$

70. $240\frac{14}{15} - 200\frac{2}{5}$

71. $15\frac{7}{8} - 8\frac{1}{2}$

72. From a lot containing $17\frac{3}{4}$ acres, $5\frac{3}{8}$ acres were sold. How many acres remained?

73. If I paid $\$4\frac{3}{10}$ for a barrel of flour, and $\$2\frac{2}{5}$ for a barrel of potatoes, how much more per barrel did the flour cost than the potatoes?

74. A man bought two suits of clothes, one costing $\$35\frac{3}{4}$, and the other $\$28\frac{1}{2}$. How much more did the one suit cost than the other?

75. If a barrel of sugar costs $\$17\frac{9}{10}$ and a barrel of pork $\$8\frac{3}{4}$, how much more does the barrel of sugar cost than the barrel of pork?

REVIEW

1. A newsboy earned $\$\frac{2}{5}$ one day, $\$\frac{3}{10}$ another day, and $\$\frac{1}{2}$ a third day. How much did he earn in the 3 days?

2. A stick was broken into two pieces — one $3\frac{3}{4}$ ft. long and the other $1\frac{3}{4}$ ft. long. How long was the whole stick?

3. If a man earns $\$3\frac{3}{5}$ a day, and a boy $\$\frac{3}{4}$ a day, how much more does the man earn in a day than the boy?

4. What will be the total cost of: 1 sack flour $\$1\frac{3}{5}$, sugar $\$\frac{1}{4}$, dried beef $\$\frac{3}{10}$, and corned beef $\$\frac{7}{20}$?

5. A boy is 4 ft. 5 in. tall. His sister is 3 ft. 5 in. tall. How much taller is the boy than his sister?

6. The top of a door is $12\frac{1}{2}$ feet above the ground, and the bottom of it is $4\frac{3}{4}$ feet above the ground. How high is the door?

7. Four loads of coal weighed as follows: 2 tons, $1\frac{3}{4}$ tons, $2\frac{1}{4}$ tons, and $2\frac{1}{2}$ tons. How much did the four loads weigh?

8. The rainfall in April was $4\frac{1}{10}$ inches, in May $3\frac{3}{4}$ inches, and in June $4\frac{1}{5}$ inches. What was the total rainfall for the three months?

9. From a barrel containing $51\frac{1}{2}$ gallons of oil, $17\frac{1}{2}$ gallons were sold in one day, and $25\frac{1}{2}$ gallons another day. How many gallons remained unsold?

10. A station agent who was paid $\$60$ per month spent in one month $\$12\frac{2}{5}$ for groceries, $\$7\frac{7}{10}$ for meat, and $\$15\frac{1}{4}$ for other expenses. How much did he save?

11. A farmer drives in one day $12\frac{3}{4}$ miles, then $6\frac{7}{8}$ miles, and then $9\frac{1}{2}$ miles. How far does he drive?

12. To the sum of $22\frac{6}{7}$ and $15\frac{3}{4}$ add their difference.

13. From $38\frac{5}{24}$ take the sum of $16\frac{7}{8}$ and $12\frac{11}{12}$.

14. The feed for a horse cost $\$5\frac{3}{4}$ per month; for a cow, $\$4\frac{1}{5}$ per month. If a man has 2 horses and 2 cows, how much will it cost to feed them a month?

15. A boy walked from his home east along a certain road $1\frac{7}{8}$ miles. He then walked to a place $2\frac{1}{4}$ miles west of his home. How far had he walked when he got home?

16. A man purchased a chair for $\$4\frac{1}{5}$, a stove for $\$6\frac{1}{2}$, a table for $\$8\frac{1}{4}$, and a bookcase for $\$12.00$. How much did they cost?

17. When a grocer receives a $10 bill in payment for sugar $3\frac{3}{4}$, vegetables $1\frac{1}{10}$, fruits $\frac{4}{5}$, rice $\frac{1}{4}$, and cakes $1\frac{1}{5}$, how much change does he give?

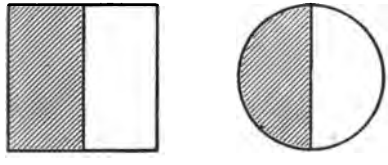
18. A student in the city in one month paid $4\frac{1}{2}\frac{1}{0}$ for car fare, $18\frac{1}{2}$ for board, $7\frac{3}{4}$ for room, $2\frac{3}{5}$ for laundry, and $6\frac{1}{2}$ for books. How much did his expenses amount to during that month?

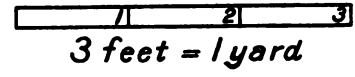
19. During the same month he earned $30 by tutoring, $18\frac{1}{2}$ by selling books, and $12\frac{9}{2}\frac{9}{0}$ by chemical work. How much did he have left after paying expenses?

20. In 3 days in June the sun shone in New York $14\frac{2}{3}$ hours, $14\frac{8}{2}\frac{8}{0}$ hours, and $14\frac{1}{3}\frac{9}{0}$ hours. How many hours of sunshine were there in these 3 days? How many hours without sunshine were there?

MULTIPLICATION OF FRACTIONS

Multiplying a fraction by a whole number.





3 feet = 1 yard

1. Into how many halves is the square divided?
2. Two times $\frac{1}{2}$ the square = —— square.
3. Two times $\$\frac{1}{2}$ = —— dollar.
4. Four times $\$\frac{1}{2}$ = —— dollar.
5. Five times $\$\frac{1}{2}$ = —— dollar.
6. Into how many feet is the yard divided?
7. What is one of these parts called?
8. What are two parts called?

9. $2 \times \frac{1}{3}$ of a yard = —— yard.

10. $6 \times \frac{1}{3}$ of a yard = —— yards.

11. $6 \times \frac{1}{2}$ of a circle = —— circles.

12. $12 \times \frac{1}{2}$ of a circle = —— circles.

13. $4 \times \frac{1}{2}$ is the same as $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$.

14. $6 \times \frac{1}{2} = \frac{6}{2}$, or 3. Therefore to multiply $\frac{1}{2}$ by 6, we say six times $\frac{1}{2} = \frac{6}{2}$, or 3.

Give products:

15.	$8 \times \frac{1}{2}$	20.	$9 \times \frac{2}{3}$	25.	$7 \times \frac{4}{9}$	30.	$3 \times \frac{7}{11}$
16.	$12 \times \frac{1}{4}$	21.	$8 \times \frac{3}{4}$	26.	$12 \times \frac{8}{9}$	31.	$12 \times \frac{4}{7}$
17.	$6 \times \frac{2}{3}$	22.	$9 \times \frac{9}{10}$	27.	$4 \times \frac{7}{11}$	32.	$5 \times \frac{7}{8}$
18.	$7 \times \frac{3}{7}$	23.	$6 \times \frac{4}{5}$	28.	$8 \times \frac{3}{7}$	33.	$6 \times \frac{4}{5}$
19.	$10 \times \frac{3}{5}$	24.	$8 \times \frac{8}{9}$	29.	$11 \times \frac{7}{8}$	34.	$6 \times \frac{8}{9}$

Finding fractional parts of whole numbers.

1. $\frac{1}{2}$ of 1 unit = —— unit.

2. $\frac{1}{2}$ of 2 units = —— unit.

3. $\frac{1}{2}$ of \$2 = —— dollar.

We have learned that we may divide a unit into any number of parts and then take any number of these parts; thus, $\frac{3}{4}$ of \$60 means that \$60 (60 units) is divided into 4 equal parts of \$15 each and that 3 of these parts, or \$45, are taken.

Since $\frac{1}{4}$ of \$60 = \$15, $\frac{3}{4}$ of \$60 = 3 × \$15 or \$45.

Find the following:

4.	$\frac{3}{4}$ of \$24.	8.	$\frac{5}{9}$ of 36 minutes.	12.	$\frac{7}{9}$ of 72 cents.
5.	$\frac{9}{10}$ of 60 horses.	9.	$\frac{11}{12}$ of \$144.	13.	$\frac{9}{11}$ of 99.
6.	$\frac{2}{3}$ of 36 days.	10.	$\frac{6}{13}$ of 78 miles.	14.	$\frac{4}{15}$ of 75.
7.	$\frac{4}{5}$ of 20 hours.	11.	$\frac{5}{11}$ of 44 rods.	15.	$\frac{3}{7}$ of 84.

16. If I wish to find $\frac{1}{3}$ of 4, into how many parts do I divide 4? How do I find one of these parts? Divide 4 by 3.

17. Explain what you mean by $\frac{1}{5}$ of a number; by $\frac{3}{4}$ of a number; by $\frac{2}{5}$ of a number.

18. Find $\frac{2}{3}$ of 4.

SOLUTION. — $\frac{1}{3}$ of $4 = \frac{4}{3}$, and $\frac{2}{3}$ of $4 = 2 \times \frac{4}{3}$ or $\frac{8}{3} = 2\frac{2}{3}$.

Find the following:

19. $\frac{2}{3}$ of 7	**25.** $\frac{9}{10}$ of 12	**31.** $\frac{3}{7}$ of 20
20. $\frac{3}{4}$ of 9	**26.** $\frac{11}{15}$ of 8	**32.** $\frac{3}{7}$ of 6
21. $\frac{7}{8}$ of 12	**27.** $\frac{7}{11}$ of 8	**33.** $\frac{4}{5}$ of 9
22. $\frac{5}{9}$ of 8	**28.** $\frac{8}{5}$ of 9	**34.** $\frac{3}{8}$ of 11
23. $\frac{6}{7}$ of 10	**29.** $\frac{4}{11}$ of 12	**35.** $\frac{7}{9}$ of 10
24. $\frac{4}{5}$ of 3	**30.** $\frac{5}{8}$ of 7	**36.** $\frac{5}{8}$ of 12

37. Multiply $\frac{3}{4}$ by 8. Which number is the multiplier? Which number is the multiplicand? Observe that $\frac{3}{4}$ of 8 gives the same result.

We find a fractional part of a number by **partition.**

If the sign of multiplication is written after a fractional multiplier, it may be read "of." Thus, $\frac{3}{4} \times 12$ may be read "$\frac{3}{4}$ of 12."

Read the following problems:

38. $\frac{2}{3} \times 9$ **39.** $\frac{3}{4} \times 6$ **40.** $\frac{7}{8} \times 4$ **41.** $\frac{6}{7} \times 6$

Written Work

1. A merchant owing $1200 gave his check for $\frac{3}{4}$ of the amount. For how much did he write his check?

2. Three men own 2500 acres of land. The first owns $\frac{2}{5}$ of it, the second $\frac{3}{5}$ of the remainder, and the third the remainder. How many acres does each own?

3. If a laborer works $\frac{4}{5}$ of the days in a common year, how many days does he work?

4. A student's expenses at college are $480 per year. If board and tuition cost $\frac{2}{3}$ of that amount, how much is spent for room rent, books, etc.?

5. A contractor agrees to erect a building for $24,570. Labor costs $\frac{1}{3}$ of the amount, material $\frac{2}{3}$ of the remainder. Find his profit.

6. $\frac{5}{8}$ of the entire enrollment of 14,720 in school are girls. Find the number of girls and the number of boys.

7. $\frac{7}{16}$ of $960 is paid in a year for rent. Find the monthly rent bill.

8. An automobile cost $3456 and the expenses and repairs for one year were $\frac{3}{16}$ of the cost. Find the expenses.

9. A western farmer bought a farm of 160 acres, at $25 an acre. He erected a house costing $\frac{8}{10}$ as much as the land, and a barn costing $\frac{1}{2}$ as much as the house. Find the total cost of the property.

Finding a number when a fractional part of it is given.

1. $\frac{2}{3}$ of a flock of sheep are 40. Find the number in the flock.

SOLUTION. — Since *two* thirds of the flock equal 40 sheep, *one* third of the flock equals $\frac{1}{2}$ of 40 sheep, or 20 sheep, and *three* thirds, or the flock, equal 3 × 20 sheep, or 60 sheep.

Find the number when:

2. $\frac{2}{3}$ of a number = 12
3. $\frac{3}{4}$ of a number = 9
4. $\frac{5}{9}$ of a number = 15
5. $\frac{7}{8}$ of a number = 21
6. $\frac{3}{7}$ of a number = 18
7. $\frac{6}{7}$ of a number = 12
8. $\frac{9}{11}$ of a number = 36
9. $12\frac{2}{13}$ of a number = 60
10. $\frac{5}{8}$ of a number = 45
11. $\frac{9}{16}$ of a number = 54

Written Work

1. There are 18 girls in a school. This number is $\frac{3}{5}$ of all the pupils in the school. How many pupils are there in the school?

2. James deposited \$18 in a savings bank, which was $\frac{3}{4}$ of what he earned during the month. How much did he earn in the month?

3. May spelled correctly 27 words, which were $\frac{9}{10}$ of all the words given. How many words were given?

4. A farmer sold 42 lambs, which were $\frac{6}{7}$ of his flock. How many lambs had he at first?

5. John has attended school 40 days, which are $\frac{2}{5}$ of the number of days in the term. Find the number of days in the term.

6. Mr. Tanner pays \$30 each quarter for his telephone. At the same rate, how much does he pay in a year?

7. \$500 is $\frac{5}{6}$ of a teacher's salary. Find her salary.

\$100 = 5 of the six parts of her salary.
1 part = \$500 ÷ 5 = \$100.
6 parts, or her salary = 6 × \$100 = \$600, salary.

8. Mr. Arnold bought a horse and a cow. He paid \$50 for the cow, which was $\frac{2}{7}$ of what the horse cost. How much did they both cost?

9. A traveling salesman drove 18 miles one day. This was $\frac{3}{11}$ as far as he rode on a train. How far did he ride on the train?

10. The cost of a barn was $\frac{2}{9}$ as much as the cost of a house. If the barn cost \$920, how much did they both cost?

11. The distance from New York to Harrisburg, Pa., is 195 miles. This is $\frac{15}{84}$ of the distance from New York to Pittsburg. What is the latter distance?

12. A steel rail is 30 feet long. This is $\frac{3}{10}$ of the length of a Pullman car. What is the length of a Pullman?

Multiplying a mixed number by an integer.

Analyze these problems:

1. Find the cost of 6 eggs at $3\frac{1}{2}$ cents a piece.

2. Find the cost of 4 qt. of oil at $4\frac{1}{2}$ cents per qt.

3. How much will 8 lb. of rice cost at $6\frac{1}{2}$ cents per lb.?

4. Find the cost of 12 lb. of sugar at $6\frac{1}{2}$ cents per lb.

5. When berries are sold for $8\frac{1}{2}$ cents per basket, find the cost of 6 baskets.

6. A man earns $\$1\frac{3}{4}$ per day. How much does he earn in 6 days?

7. When apples are sold for $\$1\frac{1}{4}$ per bushel, find the cost of 8 bushels.

8. Mary pays $\$\frac{3}{4}$ for music lessons and takes two lessons per week. How much do her music lessons cost her in 4 weeks?

9. John makes $\frac{3}{8}$ of a cent on each paper and averages 40 papers each day for six days. Find his profit.

10. We pay $7\frac{1}{2}¢$ per qt. for milk. How much is our milk bill in 4 weeks, if we use 2 qt. per day?

SUGGESTION. — Find the bill for each day; then for each week; and then for the number of weeks.

11. By buying 25 cents worth of street car tickets, each ticket costs me $4\frac{1}{6}$ cents. Find the cost of 12 tickets.

12. A storekeeper makes $1\frac{2}{3}$ cents on each can of corn. How much does he make on 18 cans?

13. John works for $5\frac{1}{2}$ cents per hour. If he works 8 hours a day, how much does he earn in 2 days?

14. Mary uses $1\frac{3}{8}$ yd. of ribbon for a bow. How much does it take for 9 such bows?

15. Find the cost of a dozen eggs at $3\frac{1}{3}$ cents apiece.

16. I pay $\$5\frac{3}{4}$ for a boy's suit. Find the cost of 4 such suits.

17. 3 baskets of cherries cost 25 cents. How much will 12 baskets cost?

SUGGESTION. — 12 baskets will cost how many times 3 baskets?

Written Work

1. Find $6 \times 1\frac{1}{2}$.

$$\begin{array}{r} 1\frac{1}{2} \\ 6 \\ \hline 3 = 6 \times \frac{1}{2} \\ 6 \\ \hline 9 \end{array}$$

This means that $6 \times \frac{1}{2}$ is to be added to 6×1.
$6 \times \frac{1}{2} = \frac{6}{2}$ or 3; $6 \times 1 = 6$; and $3 + 6 = 9$.

Find the value of:

2.	$8 \times 4\frac{1}{2}$	11.	$10 \times \$2\frac{2}{5}$	20.	$125 \times \$18\frac{3}{5}$
3.	$10 \times 4\frac{1}{2}$	12.	$12 \times \$3\frac{3}{4}$	21.	$72 \times \$24\frac{5}{9}$
4.	$9 \times 2\frac{2}{3}$	13.	$20 \times \$5\frac{2}{5}$	22.	$100 \times \$14\frac{7}{10}$
5.	$12 \times 3\frac{3}{4}$	14.	$45 \times \$12\frac{7}{9}$	23.	$132 \times \$5\frac{5}{11}$
6.	$11 \times 3\frac{3}{11}$	15.	$120 \times \$22\frac{3}{8}$	24.	$168 \times \$10\frac{5}{12}$
7.	$14 \times 2\frac{2}{7}$	16.	$154 \times \$11\frac{6}{7}$	25.	$20 \times \$18\frac{4}{5}$
8.	$12 \times 8\frac{1}{2}$	17.	$96 \times \$6\frac{3}{8}$	26.	$90 \times \$15\frac{5}{6}$
9.	$6 \times 8\frac{1}{2}$	18.	$144 \times \$9\frac{1}{6}$	27.	$50 \times \$16\frac{1}{2}$
10.	$5 \times 3\frac{3}{5}$	19.	$80 \times \$4\frac{1}{2}$	28.	$200 \times \$15\frac{3}{20}$

29. A book dealer purchases 125 books at wholesale at $\$1\frac{1}{5}$ each. Find the cost.

30. The car fare from Pittsburg to Chicago on the Ft. Wayne is $\$10\frac{1}{2}$. Find the amount received from the sale of 50 tickets.

31. A newsdealer buys 300 papers at $1\frac{1}{4}$ cents each and sells them at 2 cents each. Find the cost and the gain.

32. A huckster buys 20 dozen bananas at 10 cents per dozen and sells them at the rate of 2 cents each. Find his gain.

33. A fruit dealer buys a barrel of apples for $\$4\frac{1}{2}$. The barrel contains 240 apples, and he sells one half of them at the rate of 2 for 5 cents and the remainder at the rate of 3 for 5 cents. Find his profit.

34. A merchant buys a roll of calico containing 40 yards at $5\frac{3}{4}$ cents per yard. Find the cost.

35. The merchant retails the calico at $7\frac{1}{2}$ cents per yard. Find the amount received for the roll.

36. A boy lives $1\frac{3}{8}$ miles from his school and attends 150 days in the term. How many miles does he walk in a term, both to and from school?

37. A department store employs 100 cash girls at $\$2\frac{3}{4}$ per week and 120 other clerks at $\$4\frac{3}{4}$ per week. Find the amount paid to all.

38. If a man can cut an average of $2\frac{1}{4}$ cords of wood in a day, how many cords can he cut in 44 days?

39. A dealer in feed finds that a car load of 600 bushels of oats, after paying freight, damage, etc., costs $42\frac{7}{8}$ cents per bushel. Find the cost.

40. A contractor buys 20 thousand feet of lumber at $\$20\frac{3}{4}$ per thousand and 16 thousand bricks at $\$16\frac{1}{2}$ per thousand. Find his bill.

41. Find the cost of 24,000 railroad ties at $62\frac{1}{2}¢$ each.

42. When lead pencils are selling at $\$1\frac{1}{8}$ per gross (144), find the cost of 3550 gross.

43. Find the cost of a web of cloth containing 40 yards at $\$1\frac{7}{8}$ per yard.

44. A contractor averages $6\frac{7}{8}$ rd. a day in digging a sewer. How long is the sewer if it takes him 39 days to dig it?

45. A rural mail carrier travels $23\frac{7}{8}$ miles for each delivery. Find the number of miles traveled in 310 deliveries.

46. An ocean steamer burns on an average $201\frac{5}{16}$ tons of coal in a day. How much coal will it consume in a voyage of 7 days?

Multiplying an integer by a mixed number.

Analyze these problems:

1. Multiply 12 by $6\frac{3}{4}$; 10 by $7\frac{4}{5}$.

2. $7\frac{1}{2}$ times 8 hours are how many hours?

3. How much do $2\frac{3}{4}$ pounds of candy cost at 40 cents a pound?

4. I bought $4\frac{7}{8}$ yards of ribbon at 40 cents a yard. How much did it cost?

5. A boy walks 3 miles in an hour. How far can he walk in $2\frac{5}{6}$ hours?

6. James is 6 years old. His mother is $4\frac{5}{6}$ times as old. How old is she?

7. How much will $10\frac{3}{4}$ pounds of meat cost at 16 cents a pound?

8. A man bought $7\frac{1}{2}$ gallons of oil at $12¢$ a gallon. How much did he pay for it?

9. When gas costs 25 cents per thousand feet, what is my bill for $10\frac{3}{5}$ thousand feet?

10. If a lot cost $200 and a house $6\frac{5}{8}$ times as much, how much did the house cost?

11. A man worked $20\frac{3}{4}$ days in a month for $2 a day. How much did he earn?

12. How many inches equal $9\frac{3}{4}$ feet?

13. At 60 cents a bushel, how much will $2\frac{1}{2}$ bushels of wheat cost?

14. I bought $1\frac{3}{4}$ dozen collars at $2 per dozen. How much did they cost?

15. At 12 cents a pound, how much will $15\frac{3}{4}$ pounds of raisins cost?

16. How much will $5\frac{7}{8}$ bu. raspberries cost at $2 a bushel?

17. If a plumber is paid 75 cents per hour, how much does he receive in $3\frac{2}{5}$ hours?

18. How far will an automobile travel in $2\frac{1}{2}$ hours if it travels 18 miles in one hour?

19. If the freight from New York to Albany on a ton of merchandise is 33 cents, how much will it be on $5\frac{8}{11}$ tons?

20. A gallon of water weighs 8 pounds. How much do $10\frac{3}{4}$ gallons weigh?

Written Work

1. Multiply 12 by $18\frac{1}{6}$.

12

$18\frac{1}{6}$

2 = $\frac{1}{6} \times 12$

96

12

218

$18\frac{1}{6}$ times 12 means that $\frac{1}{6}$ of 12 is to be added to 18 × 12. $\frac{1}{6}$ of 12 = 2, which added to 18 × 12 = 218.

2. Multiply 12 by $14\frac{2}{3}$.

$$\begin{array}{r} 12 \\ 14\frac{2}{3} \\ \hline 8 = \frac{2}{3} \text{ of } 12 \\ 48 \\ 12 \\ \hline 176 \end{array}$$

$14\frac{2}{3}$ times 12 means that $\frac{2}{3}$ of 12 is to be added to 14×12. $\frac{2}{3}$ of $12 = 8$, which added to $14 \times 12 = 176$.

Find products:

3. $7\frac{1}{2} \times 6$
4. $15\frac{1}{3} \times 9$
5. $27\frac{1}{6} \times 12$
6. $120\frac{1}{8} \times 40$
7. $216\frac{1}{10} \times 50$
8. $73\frac{1}{3} \times 15$
9. $140\frac{1}{4} \times 28$
10. $100\frac{1}{10} \times 60$
11. $95\frac{1}{5} \times 45$
12. $81\frac{1}{7} \times 21$
13. $120\frac{1}{9} \times 81$
14. $144\frac{1}{2} \times 108$
15. $150\frac{1}{15} \times 60$
16. $180\frac{1}{18} \times 18$
17. $20\frac{1}{20} \times 100$
18. $42\frac{1}{11} \times 55$
19. $64\frac{1}{13} \times 39$
20. $72\frac{1}{14} \times 42$
21. $102\frac{1}{16} \times 80$
22. $124\frac{1}{24} \times 120$
23. $12\frac{2}{3} \times 9$
24. $14\frac{2}{5} \times 10$
25. $20\frac{3}{4} \times 12$
26. $35\frac{5}{6} \times 18$
27. $95\frac{1}{7} \times 42$
28. $100\frac{3}{5} \times 20$
29. $124\frac{1}{8} \times 120$
30. $65\frac{3}{8} \times 32$
31. $116\frac{2}{9} \times 54$
32. $112\frac{3}{10} \times 50$
33. $88\frac{3}{7} \times 28$
34. $30\frac{5}{8} \times 160$
35. $19\frac{7}{12} \times 24$
36. $3\frac{7}{15} \times 60$
37. $145\frac{3}{11} \times 55$
38. $48\frac{6}{7} \times 84$
39. $21\frac{3}{4} \times 16$
40. $40\frac{4}{5} \times 25$
41. $121\frac{6}{7} \times 49$
42. $10\frac{3}{8} \times 18$
43. $14\frac{7}{8} \times 24$
44. $20\frac{1}{12} \times 84$

45. If the rate of sailing of a vessel is 18 miles an hour, how far will it sail in $24\frac{1}{3}$ hours?

46. Find the cost of $12\frac{1}{4}$ tons of coal at \$6 a ton.

47. Find the cost of $16\frac{1}{2}$ yards of silk at \$1.50 a yard.

48. A farmer sold $5\frac{1}{2}$ acres of land at \$40 an acre. How much did he receive for it?

49. If ribbon is sold at 24 cents a yard, how much will $7\frac{3}{4}$ yards cost?

50. At 16 cents a pound, how much will $10\frac{7}{8}$ pounds of cheese cost?

51. If clover seed is selling at $8 a bushel, how much will $11\frac{3}{4}$ bushels cost?

52. Find the cost of $5\frac{3}{4}$ yards of point lace at $24 a yard.

53. When hay is selling for $12 a ton, how much must I pay for $16\frac{3}{4}$ tons?

54. If a boy walks 18 miles in a day, how far can he walk in $36\frac{2}{3}$ days?

55. Mr. Penrose sold his farm containing $85\frac{7}{8}$ acres at $56 an acre. How much did he receive for it?

56. Find the cost of $15\frac{3}{4}$ yards of velvet at $5 a yard.

REVIEW OF MIXED NUMBERS

1. Find the cost of 2 dozen cans of tomatoes at $8\frac{1}{3}$ cents per can.

2. How much does a motorman earn in $13\frac{1}{2}$ hours at 22¢ per hour?

3. Two men lay $29\frac{2}{3}$ ft. of sidewalk the first day, and $34\frac{1}{4}$ ft. the second day. Find the cost at 15¢ per foot.

4. A grocer sold on an average $3\frac{1}{12}$ barrels of sugar per day for 24 days. How many barrels of sugar did he sell?

5. A field containing 12 acres of wheat yielded $22\frac{3}{4}$ bushels per acre. How much was the total yield?

6. A man with a steam plow can plow $2\frac{1}{3}$ acres of ground per day. How many acres can he plow in 14 days?

7. Three men with a steam plow can plow $16\frac{1}{3}$ acres of ground per day. How many acres can they plow in 14 days?

8. An office building is 19 stories above the street. If each story averages $13\frac{1}{4}$ feet in height, how high is the building?

9. A man walked $14\frac{7}{8}$ miles a day for 18 days. How far did he walk?

10. An empty steel car weighs $18\frac{3}{4}$ tons. How much does a train of 40 such cars weigh?

11. If the average load for each car in the above train is $46\frac{7}{10}$ tons, how much does the train weigh when loaded?

12. How much will $11\frac{3}{4}$ yd. of calico cost at 8¢ a yard?

13. On Saturday a boy worked $9\frac{3}{4}$ hours at 12¢ an hour. How much did he earn?

14. During the week he worked $2\frac{1}{2}$ hours each day after school at the same rate per hour. What were his total earnings in a week?

15. If a train, including stops, runs $22\frac{3}{4}$ miles an hour, how far does it run in 7 hours?

16. An artesian well flows $19\frac{3}{8}$ gallons a minute. How many gallons flow in 40 minutes?

17. Find the cost of $3\frac{1}{2}$ dozen oranges at 24 cents per dozen.

18. A man earns $\$2\frac{1}{4}$ per day. What are his wages in 24 days?

19. A woman bought 5 pounds of sirloin steak at $16\frac{3}{4}$¢ per pound. What was the amount of her bill?

20. What is the cost of 36 dozen lead pencils at $9\frac{3}{4}$¢ per dozen?

21. When copper is selling at $16\frac{3}{4}$ cents per pound, find the cost of 36 pounds.

22. If platinum is selling at $\$19\frac{3}{4}$ per ounce, find the cost of 7 ounces.

23. Find the cost of 21 tons of anthracite coal at $\$6\frac{3}{4}$ per ton.

24. A load of soft coal contains 50 bushels. How much is it worth at $7\frac{1}{2}$ ¢ per bushel?

25. When apples are selling at \$.64 a bushel, find the cost of $12\frac{3}{4}$ bushels.

26. At $8\frac{3}{8}$ cents per pound, how much is the expressage on a package weighing 48 pounds?

27. If a bale of cotton weighs $475\frac{3}{4}$ pounds, how much is it worth at 8 cents per pound?

28. How much will $17\frac{3}{4}$ quarts of ice cream cost at 20 cents per quart?

29. When hay is selling at $\$8\frac{3}{4}$ per ton, find the cost of 34 tons.

30. From a piece of carpet containing 61 yd., $19\frac{3}{4}$ yd. were sold at 70 ¢ a yard, $17\frac{1}{2}$ yd. at 65 ¢ a yard, and the remainder at 55 ¢ a yard. For how much was the whole piece sold?

31. If the above piece of carpet cost 45 ¢ a yard, find the entire gain.

32. Find the weight of 14 bags of coffee if each bag weighs $47\frac{3}{8}$ pounds.

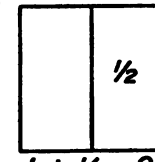
33. At $8\frac{7}{8}$ miles per hour, how far does a steamboat travel in 16 hours?

34. The rate in example 33 is the rate downstream. Upstream the rate is $2\frac{3}{4}$ miles per hour less. How far would the boat travel upstream in 24 hours?

DIVISION OF FRACTIONS

Dividing a whole number by a fraction.

1. How many halves are there in this square?

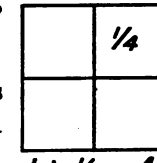


1 ÷ ½ = 2

2. How many times is $\frac{1}{2}$ contained in 1?

3. How many times $\$\frac{1}{2}$ are there in \$1? in \$2? in \$4?

4. What is the quotient of *two* halves divided by *one* half? of $\frac{2}{2} \div \frac{1}{2}$? What is the quotient of *four* halves divided by *two* halves? of $\frac{4}{2} \div \frac{2}{2}$? of $\frac{6}{2} \div \frac{2}{2}$?



1 ÷ ¼ = 4

5. How many *fourths* are there in this square? What, then, is the quotient of 1 divided by $\frac{1}{4}$?

6. How many $\$\frac{1}{4}$ are there in \$1? in \$3?

7. What is the quotient of *four* fourths divided by *one* fourth? of $\frac{4}{4} \div \frac{1}{4}$? What is the difference between the quotients of $1 \div \frac{1}{4}$ and $\frac{4}{4} \div \frac{1}{4}$? of $2 \div \frac{1}{4}$ and $\frac{8}{4} \div \frac{1}{4}$? of $3 \div \frac{1}{4}$ and $\frac{12}{4} \div \frac{1}{4}$?

8. Explain how $1 \div \frac{1}{4} = \frac{4}{4} \div \frac{1}{4}$; $2 \div \frac{1}{2} = \frac{4}{2} \div \frac{1}{2}$; $3 \div \frac{1}{4} = \frac{12}{4} \div \frac{1}{4}$; $\frac{2}{3} \div \frac{1}{4} = \frac{8}{12} \div \frac{3}{12}$; $\frac{5}{6} \div \frac{1}{5} = \frac{25}{30} \div \frac{6}{30}$.

Give quotients:

9. $2 \div \frac{1}{2}$	15. $5 \div \frac{1}{3}$	21. $12 \div \frac{1}{2}$	27. $15 \div \frac{1}{2}$
10. $4 \div \frac{1}{2}$	16. $6 \div \frac{1}{4}$	22. $10 \div \frac{1}{2}$	28. $2 \div \frac{1}{8}$
11. $5 \div \frac{1}{2}$	17. $6 \div \frac{1}{3}$	23. $9 \div \frac{1}{2}$	29. $4 \div \frac{1}{8}$
12. $2 \div \frac{1}{3}$	18. $8 \div \frac{1}{3}$	24. $6 \div \frac{1}{5}$	30. $5 \div \frac{1}{5}$
13. $4 \div \frac{1}{3}$	19. $5 \div \frac{1}{4}$	25. $12 \div \frac{1}{3}$	31. $16 \div \frac{1}{2}$
14. $3 \div \frac{1}{3}$	20. $4 \div \frac{1}{5}$	26. $4 \div \frac{1}{8}$	32. $10 \div \frac{1}{4}$

33. Change 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 each to 4ths; to 6ths; to 8ths; to 10ths. Thus, $1 = \frac{4}{4} = \frac{6}{6} = \frac{8}{8} = \frac{10}{10}$.

34. What change must be made in $\frac{4}{5}$ and $\frac{2}{3}$ before they can be added or their difference taken?

35. What change did we make in the problem $1 \div \frac{1}{4}$ before we divided 1 by $\frac{1}{4}$?

36. What change did we make in the fractions in example 8 before one fraction was divided by another fraction? This is the *first* step.

37. When the fractions have been made similar what is the *second* step in dividing the fractions? the *third* step?

Any number may be **divided by a fraction** *by changing both numbers to similar fractions and then dividing the numerators.*

Thus, $8 \div \frac{4}{5} = \frac{40}{5} \div \frac{4}{5} = 40 \div 4 = 10.$

$\frac{5}{8} \div \frac{1}{3} = \frac{15}{24} \div \frac{8}{24} = 15 \div 8 = 1\frac{7}{8}.$

38. Give quotients:

$\frac{4}{5} \div \frac{1}{5}$	$\frac{3}{12} \div \frac{1}{4}$	$\frac{20}{25} \div \frac{1}{5}$	$\frac{12}{21} \div \frac{2}{7}$
$8 \div \frac{1}{4}$	$\frac{9}{18} \div \frac{1}{6}$	$\frac{12}{15} \div \frac{1}{3}$	$\frac{4}{5} \div \frac{2}{20}$
$\frac{9}{12} \div \frac{1}{4}$	$\frac{18}{36} \div \frac{1}{4}$	$\frac{21}{35} \div \frac{1}{5}$	$\frac{1}{2} \div \frac{4}{16}$
$\frac{8}{12} \div \frac{1}{3}$	$\frac{24}{30} \div \frac{1}{5}$	$\frac{24}{30} \div \frac{1}{6}$	$\frac{9}{10} \div \frac{3}{20}$
$\frac{15}{20} \div \frac{1}{4}$	$\frac{10}{14} \div \frac{1}{7}$	$\frac{5}{7} \div \frac{5}{14}$	$\frac{7}{8} \div \frac{2}{16}$
$\frac{12}{16} \div \frac{1}{4}$	$\frac{18}{24} \div \frac{1}{4}$	$\frac{3}{4} \div \frac{3}{12}$	$\frac{2}{3} \div \frac{4}{12}$

39. Give quotients:

$4 \div \frac{1}{3}$	$\frac{3}{5} \div \frac{4}{5}$	$\frac{7}{9} \div \frac{1}{3}$	$\frac{3}{5} \div \frac{1}{4}$
$5 \div \frac{2}{3}$	$\frac{12}{16} \div \frac{1}{8}$	$\frac{1}{10} \div \frac{1}{3}$	$\frac{4}{9} \div \frac{2}{7}$
$2 \div \frac{7}{8}$	$\frac{24}{30} \div \frac{1}{6}$	$\frac{1}{3} \div \frac{1}{4}$	$\frac{1}{8} \div \frac{1}{7}$
$\frac{1}{2} \div \frac{1}{4}$	$\frac{20}{25} \div \frac{2}{5}$	$\frac{1}{4} \div \frac{1}{3}$	$\frac{8}{9} \div \frac{4}{9}$
$\frac{3}{8} \div \frac{1}{3}$	$\frac{15}{20} \div \frac{3}{4}$	$\frac{6}{7} \div \frac{2}{3}$	$\frac{3}{10} \div \frac{7}{10}$
$12 \div \frac{3}{4}$	$\frac{5}{8} \div \frac{5}{12}$	$\frac{9}{18} \div \frac{5}{6}$	$\frac{18}{36} \div \frac{3}{4}$

Dividing a whole number or a mixed number by a fraction.

An **improper fraction** is a fraction whose numerator is equal to or greater than its denominator; as $\frac{4}{4}$, $\frac{4}{3}$, $\frac{6}{6}$, $\frac{7}{6}$.

A **proper fraction** is a fraction whose numerator is less than its denominator; as, $\frac{3}{4}$, $\frac{2}{4}$, $\frac{5}{6}$.

Change $4\frac{1}{2}$, $5\frac{1}{3}$, $2\frac{2}{3}$, $5\frac{1}{5}$, $7\frac{1}{8}$, to improper fractions.

Written Work

1. Divide $4\frac{1}{2}$ by $\frac{3}{4}$.

$4\frac{1}{2} = \frac{9}{2}$
$4 \quad = \text{c. d.}$
$\frac{9}{2} \div \frac{3}{4} =$
$\frac{18}{4} \div \frac{3}{4} = 6$

Change $4\frac{1}{2}$ to the improper fraction $\frac{9}{2}$. Change $\frac{9}{2}$ and $\frac{3}{4}$ to similar fractions. $\frac{9}{2} = \frac{18}{4}$, $\frac{3}{4} = \frac{3}{4}$, $\frac{18}{4} \div \frac{3}{4} = 18 \div 3$, or 6.

Divide:

2. $6 \div \frac{1}{3}$
3. $3\frac{3}{4} \div \frac{2}{3}$
4. $5 \div \frac{1}{2}$
5. $2\frac{1}{2} \div \frac{3}{4}$
6. $2\frac{1}{8} \div \frac{1}{2}$
7. $3\frac{1}{3} \div \frac{1}{5}$
8. $1\frac{4}{5} \div \frac{9}{10}$
9. $10 \div \frac{1}{4}$
10. $8 \div \frac{2}{3}$
11. $12 \div 3\frac{1}{5}$
12. $6 \div \frac{4}{5}$
13. $1\frac{2}{3} \div \frac{2}{3}$
14. $1\frac{7}{8} \div \frac{3}{4}$
15. $2\frac{1}{5} \div \frac{3}{4}$
16. $6 \div \frac{2}{3}$
17. $3\frac{1}{2} \div \frac{7}{8}$
18. $5\frac{3}{8} \div \frac{3}{4}$
19. $8\frac{4}{9} \div 2\frac{2}{3}$

20. How much are eggs per dozen when 36 cents are paid for $2\frac{1}{4}$ dozen?

21. A man's wages amounted to 46 dollars for $9\frac{1}{5}$ days' work. How much did he receive per day if he earned this amount in $9\frac{1}{5}$ days?

22. A sum of money yielded 71 dollars interest. How many years was the money at interest if it yielded $7\frac{1}{10}$ dollars each year?

23. A merchant sold 81 cents' worth of ribbon. If he sold $6\frac{3}{4}$ yards, what was the price per yard?

Dividing a mixed number by a mixed number.

Written Work

1. Divide $1\frac{1}{2}$ by $1\frac{1}{3}$

$6 = \text{c. d.}$

$1\frac{1}{2} + 1\frac{1}{3}$

$\frac{9}{6} + \frac{8}{6} = 9 \div 8, \frac{9}{8}$ or $1\frac{1}{8}$

First change the mixed numbers to improper fractions, then make the fractions similar, and divide the numerator of the dividend by the numerator of the divisor.

2. $7\frac{1}{2} \div 2\frac{1}{2}$
3. $6\frac{2}{3} \div 1\frac{2}{3}$
4. $5\frac{1}{4} \div 1\frac{3}{4}$
5. $8\frac{2}{3} \div 2\frac{1}{6}$
6. $7\frac{1}{3} \div 1\frac{1}{6}$
7. $3\frac{1}{3} \div 2\frac{1}{2}$
8. $12\frac{1}{2} \div 6\frac{1}{4}$
9. $14\frac{2}{7} \div 2\frac{2}{7}$
10. $3\frac{1}{4} \div 2\frac{1}{2}$
11. $5\frac{2}{3} \div 4\frac{1}{5}$
12. $6\frac{1}{8} \div 7\frac{1}{4}$
13. $8\frac{1}{3} \div 9\frac{1}{6}$
14. $3\frac{1}{6} \div 9\frac{1}{2}$
15. $3\frac{3}{7} \div 3\frac{1}{7}$
16. $8\frac{3}{5} \div 6\frac{1}{7}$
17. $9\frac{1}{6} \div 3\frac{1}{2}$
18. $3\frac{9}{10} \div 2\frac{3}{5}$
19. $6\frac{1}{2} \div 4\frac{1}{6}$

20. At $2\frac{5}{8}$ dollars each, how many vests can be bought for \$42?

21. At $\$1\frac{1}{2}$ apiece, how many pictures can be bought for $\$10\frac{1}{2}$?

22. I paid $\$28\frac{4}{5}$ for cloth at $\$1\frac{1}{5}$ a yard. How many yards did I buy?

23. I spent $\$17\frac{3}{5}$ for molasses at $\$1\frac{1}{10}$ a gallon. How many gallons did I buy?

24. At $\$1\frac{1}{5}$ each, how many straw hats can be bought for $\$14\frac{2}{5}$?

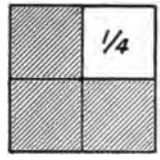
25. A merchant purchases $\$50\frac{2}{5}$ worth of gas lamps at $\$4\frac{1}{5}$ a piece. Find the number purchased.

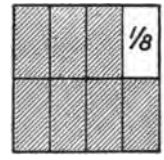
26. When $3\frac{1}{4}$ bushels of apples cost $\$2\frac{7}{16}$, how much will one bushel cost?

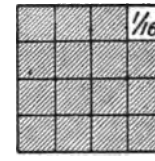
27. A man earns $\$16\frac{1}{2}$ in $5\frac{1}{2}$ days. How much is this per day?

28. At $\$2\frac{1}{2}$ per pair, how many pairs of shoes can be bought for $\$17\frac{1}{2}$?

Dividing a fraction by a whole number.







1. How many fourths of a square are there in one square?

2. How many eighths of a square are there in one square?

3. How many sixteenths of a square are there in one square?

4. $\frac{1}{2}$ of $\frac{4}{4}$ of the square $= \frac{?}{4}$ of the square.

5. In dividing $\frac{4}{4}$ by 2, we separate $\frac{4}{4}$ into two equal parts; thus, $\frac{2}{4}$, $\frac{2}{4}$, and take one of these equal parts. Observe that taking $\frac{1}{2}$ of $\frac{4}{4}$ and dividing $\frac{4}{4}$ by 2 gives the same result.

6. $\frac{1}{4}$ of $\frac{4}{8}$ of the square $= \frac{?}{8}$ of the square; then $\frac{4}{8} \div 4 = \frac{?}{8}$.

7. In dividing $\frac{8}{8}$ by 4, we separate $\frac{8}{8}$ into 4 equal parts; thus, $\frac{2}{8}$, $\frac{2}{8}$, $\frac{2}{8}$, $\frac{2}{8}$, and take one of these equal parts. Then $\frac{8}{8} \div 4 = \frac{?}{8}$. Observe that taking $\frac{1}{4}$ of $\frac{8}{8}$ and dividing $\frac{8}{8}$ by 4 gives the same result.

8. $\frac{1}{8}$ of $\frac{16}{16}$ of the square $= \frac{?}{16}$ of the square; then $\frac{16}{16} \div 8 = \frac{?}{16}$. Observe that taking $\frac{1}{8}$ of $\frac{16}{16}$ and dividing $\frac{16}{16}$ by 8 gives the same result.

9. $\frac{16}{16} \div 8 = \frac{?}{16}$

10. $\frac{12}{12} \div 3 = \frac{?}{12}$

11. $\frac{20}{20} \div 5 = \frac{?}{20}$

12. $\frac{12}{15} \div 4 = \frac{?}{15}$

13. $\frac{18}{25} \div 9 = \frac{?}{25}$

14. $\frac{30}{35} \div 6 = \frac{?}{35}$

15. $\frac{12}{18} \div 4 = \frac{?}{18}$

16. $\frac{16}{24} \div 8 = \frac{?}{24}$

17. $\frac{21}{25} \div 7 = \frac{?}{25}$

18. $\frac{14}{20} \div 2 = \frac{?}{20}$

19. $\frac{12}{30} \div 6 = \frac{?}{30}$

20. $\frac{26}{28} \div 5 = \frac{?}{28}$

21. How may a fraction be divided by a whole number?

A **fraction** *may be* **divided by a whole number** *by dividing the numerator of the fraction by the whole number.*

22. If $\frac{1}{2}$ of a square is divided into 2 equal parts, what fraction of the square is each part? Then $\frac{1}{2} \div 2 = \frac{1}{?}$.

23. If $\frac{1}{4}$ of a square is divided into 2 equal parts, what fraction of the square is each part? Then $\frac{1}{4} \div 2 = \frac{1}{?}$.

24. If $\frac{1}{2}$ of a square is divided into 8 equal parts, what fraction of the square is each part? Then $\frac{1}{2} \div 8 = \frac{1}{?}$.

25. Since $\frac{1}{2} \div 2 = \frac{1}{4}$, $\frac{1}{4} \div 2 = \frac{1}{8}$, and $\frac{1}{2} \div 8 = \frac{1}{16}$, how may we divide a fraction by a whole number without dividing the numerator by the whole number?

A **fraction** *may be* **divided by a whole number** *by multiplying the denominator by the whole number.*

26. In $\frac{6}{8} \div 3$ is it easier to divide the numerator or to multiply the denominator? in $\frac{6}{8} \div 5$?

27. State the easier method of dividing: $\frac{3}{4}$ by 3; $\frac{3}{4}$ by 2; $\frac{7}{8}$ by 2; $\frac{7}{8}$ by 7; $\frac{8}{9}$ by 4; $\frac{8}{9}$ by 5.

28. Under what conditions is it easier to divide the numerator? when is it easier to multiply the denominator?

Observing the two methods, divide the following fractions, using the method best adapted to the problem:

29. $\frac{6}{8} \div 3$	**37.** $\frac{5}{9} \div 4$	**45.** $\frac{27}{30} \div 9$
30. $\frac{12}{15} \div 4$	**38.** $\frac{8}{9} \div 4$	**46.** $\frac{30}{40} \div 6$
31. $\frac{7}{8} \div 5$	**39.** $\frac{6}{10} \div 5$	**47.** $\frac{7}{8} \div 4$
32. $\frac{8}{11} \div 3$	**40.** $\frac{24}{25} \div 6$	**48.** $\frac{9}{10} \div 5$
33. $\frac{3}{4} \div 10$	**41.** $\frac{20}{30} \div 10$	**49.** $\frac{7}{12} \div 3$
34. $\frac{5}{12} \div 6$	**42.** $\frac{35}{40} \div 7$	**50.** $\frac{12}{19} \div 6$
35. $\frac{8}{9} \div 3$	**43.** $\frac{3}{7} \div 2$	**51.** $\frac{12}{19} \div 5$
36. $\frac{7}{12} \div 2$	**44.** $\frac{10}{15} \div 5$	**52.** $\frac{15}{16} \div 4$

Written Work

1. Divide $\frac{24}{25}$ by 6.

$$\frac{24}{25} \div 6 = \frac{24 \div 6}{25} = \frac{4}{25}$$

Since dividing the numerator of a fraction divides the fraction, to divide $\frac{24}{25}$ by 6, divide the numerator by 6.

2. Divide $\frac{5}{6}$ by 4.

$$\frac{5}{6} \div 4 = \frac{5}{6 \times 4} = \frac{5}{24}$$

Since multiplying the denominator of a fraction divides the fraction, to divide $\frac{5}{6}$ by 4, multiply the denominator by 4.

3. If 8 yards of ribbon cost $\$\frac{16}{25}$, find the cost of 1 yard.

4. I bought 6 gallons of oil for $\$\frac{9}{10}$. What was the price per gallon?

5. A boy earns $\$\frac{4}{5}$ in a day by working 9 hours. How much does he earn in an hour?

6. If 5 dozen bananas cost $\$\frac{3}{4}$, how much does 1 dozen cost?

7. Eight men in a day complete $\frac{2}{13}$ of a piece of work. How much of the work could one man complete in a day?

8. It takes $\frac{3}{4}$ of a thousand feet of lumber to make 12 boxes. How many feet are there in each box?

9. If 12 panes of glass cost $\$\frac{3}{5}$, how much does one pane cost?

10. If 17 handkerchiefs cost $\$\frac{17}{20}$, how many cents does each cost?

11. In $\frac{3}{4}$ of a day a man earns $9. How long does it take him to earn $1?

12. In $\frac{11}{12}$ of an hour an oil well produces 33 barrels of oil. How long does it take it to produce one barrel?

Dividing any number by a fraction by inverting the terms of the divisor.

1. $1 \div \frac{1}{2} =$ —— $1 \div \frac{1}{4} =$ —— $1 \div \frac{1}{5} =$ ——

2. $1 \div \frac{1}{6} =$ —— $1 \div \frac{1}{7} =$ —— $1 \div \frac{1}{8} =$ ——

Observe that in each of the examples given above the quotient equals $1 \times$ the denominator, or $1 \times \frac{2}{1}$, $1 \times \frac{4}{1}$, $1 \times \frac{5}{1}$, $1 \times \frac{6}{1}$, $1 \times \frac{7}{1}$, $1 \times \frac{8}{1}$.

3. Divide 12 successively by 2, 3, 4, 6. The *quotients*, 6, 4, 3, 2 respectively, become *smaller* as the *divisors* become *larger*. Then if $1 \div \frac{1}{4} = \frac{4}{1}$ will the quotient of $1 \div \frac{3}{4}$ be larger or smaller than $\frac{4}{1}$? Since $\frac{3}{4}$ is 3 times as large as $\frac{1}{4}$, the quotient of $1 \div \frac{3}{4}$ will be $\frac{1}{3}$ of the quotient of $1 \div \frac{1}{4}$; $\frac{1}{3}$ of $\frac{4}{1} = \frac{4}{3}$.

4. $1 \div \frac{1}{5} = \frac{5}{1}$, and $1 \div \frac{2}{5} = \frac{1}{2}$ of $\frac{5}{1} = \frac{5}{2}$. $1 \div \frac{1}{6} = \frac{6}{1}$, and $1 \div \frac{5}{6} = \frac{1}{5}$ of $\frac{6}{1} = \frac{6}{5}$. Find the quotients of $1 \div \frac{4}{5}$; $1 \div \frac{7}{8}$; $1 \div \frac{9}{10}$; $1 \div \frac{3}{8}$. Notice that in every case the quotient equals the divisor with its terms inverted.

The number of times a fraction is contained in 1 equals the denominator of the fraction divided by the numerator, or **the fraction inverted.**

Find quotients by inverting the divisor and multiplying:

Thus: $3 \div \frac{1}{3} = 3 \times \frac{3}{1} = \frac{15}{4} = 3\frac{3}{4}$.

5. $1 \div \frac{1}{4}$	12. $2 \div \frac{2}{3}$	19. $8 \div \frac{7}{8}$	26. $15 \div \frac{3}{4}$
6. $1 \div \frac{2}{4}$	13. $3 \div \frac{3}{4}$	20. $16 \div \frac{3}{4}$	27. $16 \div \frac{2}{3}$
7. $1 \div \frac{2}{3}$	14. $4 \div \frac{4}{5}$	21. $24 \div \frac{4}{5}$	28. $10 \div \frac{4}{9}$
8. $1 \div \frac{7}{8}$	15. $5 \div \frac{5}{6}$	22. $20 \div \frac{4}{5}$	29. $8 \div \frac{7}{12}$
9. $1 \div \frac{3}{4}$	16. $6 \div \frac{7}{8}$	23. $15 \div \frac{3}{5}$	30. $7 \div \frac{8}{15}$
10. $1 \div \frac{3}{5}$	17. $9 \div \frac{9}{10}$	24. $30 \div \frac{2}{3}$	31. $6 \div \frac{5}{9}$
11. $1 \div \frac{5}{6}$	18. $10 \div \frac{4}{9}$	25. $12 \div \frac{5}{6}$	32. $9 \div \frac{7}{9}$

Written Work

1. Divide 128 by $\frac{16}{25}$.

$$128 \div \frac{16}{25} = \overset{8}{\cancel{128}} \times \frac{25}{\cancel{16}} = 200$$

Since $\frac{16}{25}$ is contained in 1, $\frac{25}{16}$ times, $\frac{16}{25}$ is contained in 128, $128 \times \frac{25}{16}$ or 200 times.

Any number may be **divided by a fraction** *by inverting the terms of the divisor and multiplying.*

Divide:

2. 18 by $\frac{2}{3}$	9. 63 by $\frac{7}{8}$	16. 288 by $\frac{24}{5}$
3. 25 by $\frac{5}{8}$	10. 72 by $\frac{8}{9}$	17. 400 by $\frac{16}{9}$
4. 28 by $\frac{7}{9}$	11. 84 by $\frac{21}{25}$	18. 285 by $\frac{15}{9}$
5. 21 by $\frac{3}{7}$	12. 90 by $\frac{15}{16}$	19. 546 by $\frac{21}{35}$
6. 36 by $\frac{9}{10}$	13. 108 by $\frac{12}{13}$	20. 425 by $\frac{25}{12}$
7. 42 by $\frac{6}{11}$	14. 84 by $\frac{7}{12}$	21. 378 by $\frac{14}{15}$
8. 54 by $\frac{6}{7}$	15. 96 by $\frac{8}{9}$	22. 324 by $\frac{9}{14}$

23. Divide 36 by $3\frac{3}{5}$.

SUGGESTION. — Change the divisor to an improper fraction.

Divide:

24. 27 by $2\frac{1}{4}$	30. 84 by $4\frac{1}{5}$	36. 780 by $7\frac{4}{5}$
25. 33 by $3\frac{2}{3}$	31. 75 by $2\frac{7}{9}$	37. 355 by $8\frac{7}{8}$
26. 44 by $4\frac{2}{5}$	32. 90 by $3\frac{3}{4}$	38. 295 by $6\frac{5}{9}$
27. 60 by $3\frac{3}{4}$	33. 92 by $2\frac{3}{10}$	39. 748 by $3\frac{3}{8}$
28. 76 by $4\frac{3}{4}$	34. 85 by $1\frac{3}{9}$	40. 549 by $8\frac{5}{7}$
29. 60 by $6\frac{2}{3}$	35. 245 by $5\frac{5}{6}$	41. 620 by $7\frac{3}{4}$

Divide:

42. $11\frac{3}{5}$ by $3\frac{2}{9}$	45. $4\frac{8}{15}$ by $1\frac{5}{12}$	48. $10\frac{4}{5}$ by $2\frac{4}{7}$
43. $6\frac{3}{8}$ by $1\frac{3}{9}$	46. $7\frac{7}{12}$ by $1\frac{3}{4}$	49. $15\frac{5}{8}$ by $2\frac{7}{9}$
44. $10\frac{6}{9}$ by $2\frac{2}{11}$	47. $7\frac{3}{5}$ by $2\frac{3}{8}$	50. $12\frac{3}{4}$ by $5\frac{2}{3}$

COMPARISON — WHOLE NUMBERS AND FRACTIONS

1. $\frac{1}{2} = \frac{?}{8}$; $\frac{2}{4} = \frac{?}{8}$. Then how do $\frac{1}{2}$ and $\frac{2}{4}$ compare?

2. If a unit is first divided into halves and then each half into halves, into how many parts is the unit divided? Is $\frac{1}{4}$ of a unit larger or smaller than $\frac{1}{2}$ of a unit?

3. Divide a unit into halves, fourths, eighths, and sixteenths, and show how many sixteenths of a unit it takes to make $\frac{1}{4}$ of the unit; $\frac{1}{8}$ of the unit; $\frac{3}{4}$ of the unit; $\frac{1}{2}$ of the unit.

4. Draw equal squares to show that $\frac{1}{2} = \frac{4}{8}$ or $\frac{8}{16}$.

5. Compare $\frac{8}{2}$ and $\frac{4}{2}$; $\frac{4}{5}$ and $\frac{12}{5}$; $\frac{8}{3}$ and $\frac{2}{3}$.

6. How does $\frac{4}{5}$ of 20 compare with $\frac{1}{2}$ of 20? $\frac{7}{8}$ of 16 with $\frac{3}{4}$ of 16? $\frac{3}{5}$ of 50 with $\frac{1}{4}$ of 20?

7. A has 40 acres of land, and B 60 acres. How does A's farm compare in size with B's?

8. 8 is what part of 12, 16, 24, 32, 48, 72?

SUGGESTION. — Make 8 the numerator in each case and the other numbers the denominators and reduce the fractions to their lowest terms. Thus $\frac{8}{12} = \frac{2}{3}$; $\frac{8}{16} = \frac{1}{2}$; $\frac{8}{24} = \frac{1}{3}$, etc.

9. If 5 quarts of milk cost 45 cents, how much will 12 quarts cost? 18 quarts? 24 quarts?

10. Elizabeth buys $3\frac{1}{2}$ yards of ribbon for 35 cents. At the same rate, how much would she pay for $10\frac{1}{2}$ yards?

SUGGESTION. — How many times $3\frac{1}{2}$ is $10\frac{1}{2}$?

11. A woodsman cuts 15 cords of wood in 6 days. How many cords, at the same rate, could he cut in 48 days?

12. Compare 8 and 2; 6 and 4; 2 and 8; 5 and 10.

13. Compare $\frac{1}{2}$ and $\frac{1}{4}$; $\frac{1}{4}$ and $\frac{1}{2}$; $\frac{1}{6}$ and $\frac{1}{12}$.

14. Compare 24 with 4, 6, 8, 48, 72, 16, 20.

15. Draw a square inch and show that $\frac{1}{4}$ of it $= \frac{4}{16}$ of it; that $\frac{1}{2}$ of it $= \frac{8}{16}$ of it.

16. If $\frac{1}{4}$ of a man's weekly wages is \$2.75, how much is $\frac{1}{2}$ of his weekly wages?

SUGGESTION. — $\frac{1}{4}$ is how many times 1?

17. $3\frac{1}{2}$ pounds of rice cost 35 cents. At that rate how much will 7 pounds cost?

18. If $\frac{1}{2}$ of my money is \$10, and George has 6 times my money, how much has George?

19. In New York $9\frac{1}{4}$ inches of rain fell in 4 months. At that rate how much will fall in a year?

20. If a man pays \$3675 for 60 acres of land, at the same rate, how much should he pay for 120 acres?

21. My telephone bill is \$12.85 a month. At that rate how much should I pay in $2\frac{3}{4}$ years?

22. My coal bill for $5\frac{1}{2}$ tons is \$11. What is the bill of my neighbor who buys $27\frac{1}{2}$ tons at the same rate?

23. If 30 bushels of oats sell for \$13.20, how much will 60 bushels sell for?

24. If a boy receives \$7.50 for two weeks' work, how much should he receive for 12 weeks' work?

25. How much will a clerk earn in a year if he earns \$180 in 3 months?

26. If 4 tons of coal cost $\$8\frac{4}{5}$, how much will 16 tons cost?

27. When 5 books cost \$17.50, how much will 25 such books cost?

28. A man walked $11\frac{1}{4}$ miles in 3 hours. At the same rate, how far would he walk in 15 hours?

REVIEW OF FRACTIONS

1. A boy earns $\$2\frac{3}{4}$ a week and spends $\$1\frac{5}{8}$. If he puts the remainder in bank, what will his bank account be in 19 weeks?

2. May went to the store with $\$20$. If she spends $\$2\frac{1}{2}$ for a pair of shoes and $\$4\frac{3}{4}$ for a hat, what change will she receive?

3. From a web of muslin containing 60 yards, $19\frac{3}{4}$ yd. were sold to one customer, $13\frac{1}{2}$ yd. to another, and $9\frac{1}{4}$ yd. to another. How many yards remained?

4. A dealer sold 2 loads of coal at $\$10\frac{2}{3}$ a ton. If the first load contained $2\frac{3}{8}$ T., and the second $3\frac{1}{4}$ T., how much did he receive?

5. How many yards of muslin can be bought for $75\,¢$, if 3 yards cost $25\,¢$?

6. If John weighs $96\frac{3}{4}$ lb. and Mary weighs $36\frac{1}{2}$ lb. less than John, how much do they both weigh?

7. A farmer bought 480 acres of land. He sold $148\frac{1}{9}$ acres to one man, and $125\frac{1}{3}$ acres to another. What was the remainder worth at $\$20$ an acre?

8. How many days will it take a man to earn $\$126\frac{1}{2}$, if he earns $\$2\frac{3}{4}$ a day?

9. A man dies leaving $\$4440$. He leaves $\frac{1}{12}$ of it to his son, $\frac{1}{3}$ of it to his daughter, $\frac{2}{3}$ of it to his widow, and the rest to a hospital. How much does each receive?

10. John bought 3 pieces of cloth containing $19\frac{1}{2}$ yd., $26\frac{1}{4}$ yd., and $35\frac{3}{8}$ yd. respectively. How many yards did he buy?

11. A man worked $43\frac{3}{4}$ hr. one week, and $56\frac{5}{16}$ hr. the next. How many hours did he work in the two weeks?

12. Mr. Jones one day spent 3 half dollars, 7 quarters, 15 nickels, and 3 cents. How much did he spend?

13. A jeweler bought 365 clocks for $4380, and sold them so as to gain $3 on each one. What was the selling price per clock?

14. A dealer sold 165 horses for $7425, which sum was $495 more than they had cost him. What was the average cost of each horse?

15. Mrs. Smith bought 4 pieces of lace containing $4\frac{3}{8}$ yd., $6\frac{3}{4}$ yd., $5\frac{1}{2}$ yd., and $9\frac{7}{8}$ yd., respectively. How much were they worth at $12\frac{1}{2}$ ¢ a yard?

16. If an acre of land is worth $32, how much is $\frac{3}{4}$ of an acre worth?

17. A man paid $45 for $\frac{5}{8}$ of a quantity of grain. Find the value of the entire quantity at the same rate.

18. How many poor families can be supplied with $\frac{1}{8}$ of a ton of coal each, from $9\frac{7}{8}$ tons?

19. Find the cost of $36\frac{1}{4}$ pounds of tea at 32¢ a pound.

20. If $\frac{3}{4}$ of a yard of silk cost 45 cents, what is the value of $6\frac{1}{2}$ yards at the same rate?

21. What number taken from the sum of $16\frac{1}{2}$ and $28\frac{2}{3}$ will leave $19\frac{1}{4}$?

22. What is the cost of $\frac{3}{4}$ of a yard of muslin at 12¢ a half yard?

23. The sum of two numbers is $126\frac{2}{3}$, and the larger number is $94\frac{3}{4}$. What is the smaller number?

24. A man spends $6\frac{1}{2}$ for board, $12\frac{1}{4}$ for clothing, $5\frac{1}{6}$ for books, and has $12 left. How many dollars and cents had he at first?

25. If 75¢ is the cost of $\frac{3}{4}$ of a yard of cloth, what is the cost of 5 yards?

26. \$1.50 is the value of $\frac{5}{8}$ of a yard of broadcloth. How many yards can be bought for \$20?

27. $\frac{3}{4}$ of a number is 9. What is the number?

28. In traveling 72 miles a man went $\frac{2}{3}$ of the distance the first day, $\frac{1}{8}$ of the distance the second day, and the remainder the third day. How far did he travel the third day?

29. From a farm of $225\frac{3}{4}$ acres there were sold $150\frac{1}{2}$ acres. How many acres were left?

30. How much must be paid for 16 tables at $\$7\frac{3}{4}$ each?

31. At $\$\frac{3}{4}$ each, how many books can I buy for \$36?

32. A man bought a hat for $\$5\frac{3}{4}$, a pair of shoes for $\$6\frac{1}{2}$, a pair of gloves for $\$3\frac{1}{2}$, and a suit for $\$22\frac{1}{4}$. How much did he pay for all?

33. A clerk earns \$75 a month and spends $\$36\frac{3}{4}$. How much has he left?

34. At $\$.87\frac{1}{2}$ a bushel, what is the cost of 56 bushels of grain?

35. How many chairs, at $\$5\frac{1}{4}$ each, can be bought for $\$262\frac{1}{2}$?

36. A lady bought $\frac{3}{4}$ of a yard of narrow lace, $1\frac{5}{8}$ yards of medium width ribbon, and $2\frac{1}{2}$ yards of wide lace. How many yards did she buy in all?

37. From a piece of muslin containing 40 yards there were sold $25\frac{7}{8}$ yards. How many yards remained?

38. At $\$.12\frac{1}{2}$ a yard, what is the cost of 42 yd. of muslin?

39. How many pencils, at $\frac{1}{2}$¢ each, can be bought for \$1.16?

40. One coat requires $2\frac{3}{8}$ yards. How many coats can be made out of $35\frac{5}{8}$ yards?

41. John weighs $124\frac{1}{2}$ pounds, and Ned weighs $18\frac{3}{4}$ pounds less. How many pounds does Ned weigh?

42. A farmer sold $35\frac{3}{4}$ bushels at one time, and $21\frac{7}{8}$ bushels at another time. How many bushels did he sell?

43. A tailor uses $9\frac{3}{4}$ yards of cloth for a suit. How many yards will it take for 32 suits?

44. John and James cut $3\frac{3}{8}$ cords of wood in one day. In how many days can they cut $84\frac{3}{4}$ cords?

45. A man raised $236\frac{1}{2}$ bushels of oats, and sold $129\frac{3}{4}$ bushels. How many bushels had he left?

46. A man exchanged with a grocer $2\frac{1}{4}$ bushels of potatoes at 60¢ a bushel, for $6\frac{3}{4}$ pounds of bacon. How much did the grocer charge per pound for the bacon?

47. A merchant sold some grain for \$63 which was $\frac{7}{9}$ of its cost. What was the cost of the grain?

48. $\frac{3}{5}$ of a building is valued at \$300? What is the value of the whole building?

49. $1\frac{1}{2}$ lb. of butter cost 30¢. Find the cost of $2\frac{3}{4}$ lb.

50. A owns 160 sheep and B owns $\frac{1}{4}$ as many as A. C owns $\frac{1}{2}$ as many as B. How many sheep do all own?

51. Show how the denominators of fractions may be changed to denominations of higher value.

52. How many potatoes are there in 12 barrels, if one barrel contains 2 bushels 3 pecks?

53. $48 - 12\frac{1}{2} = ?$ $12\frac{3}{4} \div 5\frac{2}{5} = ?$ $10\frac{5}{8} \div 1\frac{3}{8} = ?$

54. For 5 cents one can buy 3 oranges. How much must be paid for 12 oranges?

DECIMALS

Decimal parts of the dollar.

1. How many dimes equal a dollar? Then what part of a dollar is a dime?

2. How many cents equal a dollar? Then what part of a dollar is a cent?

3. Ten mills equal one cent. How many mills equal a dollar? (Then what part of a dollar is a mill?)

Mills are not coined, but are used for exactness in computations.

When we think of a dollar as dimes, it has 10 equal parts; when we think of a dollar as cents, it has 100 equal parts; when we think of a dollar as mills, it has 1000 equal parts. A mill is $\frac{1}{10}$ of a cent; a cent $\frac{1}{10}$ of a dime; and a dime $\frac{1}{10}$ of a dollar.

This division of the dollar into tenths, hundredths, thousandths, etc., we call **decimal parts of the dollar.**

The **decimal point** is the point separating dollars and cents. Thus in \$2.75 the point separates 2 dollars from 75 cents.

4. What decimal part of a dollar are 5 dimes? 6 dimes? 8 dimes? 9 dimes?

5. What decimal part of a dollar are 5 cents? 8 cents? 9 cents? 10 cents?

The first place to the right of the decimal point is occupied by *dimes* or *tenths* of a dollar; the second place, by *cents* or *hundredths* of a dollar; the third place, by *mills* or *thousandths* of a dollar.

Since 8 dimes and 5 cents = 85 cents, we generally say that cents occupy the first two places to the right of the decimal point. Observe that dimes, cents, and mills can always be written as decimal parts of a dollar; thus, 8 dimes = \$.80; 2 mills = \$.002.

6. In \$1.256, state what each figure represents.

7. Name the parts of a dollar, first as tenths, hundredths, and thousandths; then as cents and mills: \$.65, \$8.05, \$2.005, \$.50, \$.75, \$.80, \$.705.

8. Write in figures: six dollars and five cents; ten dollars and fifty cents; three mills; five cents; five mills.

We may also find tenths, hundredths, thousandths, etc., of any unit.

READING AND WRITING DECIMALS

One tenth may be written .1 as well as $\frac{1}{10}$; **one hundredth** may be written .01 as well as $\frac{1}{100}$; and **one thousandth** may be written .001 as well as $\frac{1}{1000}$.

1. Read: .8 ft., .5 lb., .7 pk., .5 ft., .7 mi.

A **decimal point** is a period placed before tenths.

A **decimal fraction** is any number of 10ths, 100ths, 1000ths, etc., of a unit. When expressed after a decimal point and without a written denominator it is usually called a **decimal.**

The first place to the right of the decimal point is called **tenths,** the second place **hundredths,** and the third place **thousandths.**

2. In 55.55, the 5 hundredths is what part of the 5 tenths? the 5 tenths is what part of the 5 units? the 5 units is what part of the 5 tens?

In any number, whether a whole number or a decimal, *the value of a figure in any place is $\frac{1}{10}$ of the value of the same figure standing one place to the left.*

3. What is the largest decimal division of a unit? the second largest? the third largest?

4. $.06 = \frac{?}{100} = \frac{?}{1000}$

5. $.25 = \frac{?}{100} = \frac{?}{1000}$

6. $.05 = \frac{?}{100} = \frac{?}{1000}$

7. $.9 = \frac{?}{10} = \frac{?}{100} = \frac{?}{1000}$

8. $.025 = \frac{?}{1000} = \frac{?}{10000}$

9. $.349 = \frac{?}{1000} = \frac{?}{10000}$

Observe that a decimal is always less than a unit.

Hundreds	Tens	Ones	Dec. Point	Tenths	Hundredths	Thousandths
5	2	5	·	2	5	6

This number is read, five hundred twenty-five *and* two hundred fifty-six thousandths.

10. What do we call the decimal point when we read a number? What word, then, always joins the whole number and the decimal?

Observe that we express every number as units, or ones, and parts of a unit. Thus: 525 units and .256 of a unit.

As the first decimal division of a unit is tenths, we always begin to enumerate the decimal at tenths' place; thus:

tenths	hundredths	thousandths
.0	0	5

11. At what place do we begin to enumerate whole numbers?

12. Read the following: .25, .025, 25.005, 7.05, 321.1, 0.875, 1.008, 100.001, 0.001.

13. Write as decimals: $\frac{5}{10}$, $\frac{7}{100}$, $\frac{25}{1000}$, $\frac{1}{10}$, $\frac{15}{1000}$, $\frac{2}{1000}$, $26\frac{6}{1000}$, $100\frac{1}{1000}$, $1\frac{8}{100}$, $70\frac{105}{1000}$.

Write decimally:

14. Two thousandths.

15. Two and two thousandths.

16. Five hundredths.

17. Two hundred and two thousandths.

18. Two hundred two thousandths.

19. Three and five tenths.

20. Seventy-five hundredths.

21. Five hundred and five thousandths.

22. Thirty-three thousandths.

23. Ninety-five thousandths.

24. Two hundred and five hundredths.

25. Six and nine tenths.

26. Six hundred and six hundredths.

COMPARISON OF COMMON FRACTIONS AND DECIMALS

1. $\frac{5}{10} = \frac{50}{100} = \frac{500}{1000}$; $.5 = .50 = .500$.

2. Do naughts at the right of a decimal affect its value? *Annexing naughts to the right of a decimal does not affect its value.*

3. What is a fractional unit?

4. What is the largest fractional unit that may be expressed decimally? the second largest? the third largest?

5. Change $\frac{50}{100}$ to tenths.

6. Express .25, .45, .75, .025, each in the form of a common fraction.

7. Change .5 to equivalent decimals expressed in hundredths and thousandths. Thus .5 = .50 = .500.

8. Name the three largest fractional units in their order; the three largest fractional units expressed decimally.

9. Change $\frac{125}{1000}$ to an equivalent decimal.

Changing a decimal to a common fraction.

1. Change .75 to a common fraction in its lowest terms.

$.75 = \frac{75}{100} = \frac{3}{4}$

Expressed in the form of a common fraction $.75 = \frac{75}{100}$. By dividing both numerator and denominator of $\frac{75}{100}$ by 25, we reduce it to its lowest terms, $\frac{3}{4}$.

To change a decimal to a common fraction, write the decimal, omitting the decimal point, place the decimal denominator beneath it, and change the fraction to its lowest terms.

Change to fractions:

2. .15	4. .9	6. .75	8. .125
3. .825	5. .325	7. .025	9. .425

10. Memorize the following equivalents:

$\frac{1}{2} = .5$ or .50	$\frac{1}{5} = .2$ or .20	$\frac{4}{5} = .8$ or .80
$\frac{1}{4} = .25$	$\frac{2}{5} = .4$ or .40	$\frac{1}{8} = .125$
$\frac{3}{4} = .75$	$\frac{3}{5} = .6$ or .60	$\frac{3}{8} = .375$

11. Change to *tenths*: $\frac{1}{5}$; $\frac{1}{2}$; $\frac{2}{5}$; $\frac{3}{5}$; $\frac{4}{5}$.

12. Express as decimal hundredths: $\frac{1}{4}$; $\frac{3}{4}$; $\frac{1}{8}$; $\frac{3}{8}$.

Change to fractions and reduce to lowest terms:

13. .45	19. .20	25. .40	31. .075
14. .625	20. .60	26. .48	32. .025
15. .75	21. .125	27. .150	33. .08
16. .65	22. .90	28. .50	34. .225
17. .375	23. .96	29. .025	35. .700
18. .80	24. .72	30. .08	36. .800

ADDITION OF DECIMALS

1. What kind of fractions can be added or subtracted?

In *adding* or *subtracting* decimals, like units must always be written under one another; thus, .8 + .85 + .096 may be written thus:

	.8		.8 + .8 = 1.6
Added	.85	**Test:**	.05 + .09 = .14
	.096		.006 = .006
	1.746		1.746

2. In how many of the decimals were there tenths to be added? hundredths? thousandths?

3. Why must tenths be written *under* tenths, hundredths *under* hundredths, etc.?

A **mixed** decimal is a whole number and a decimal united; thus, 4 + .05 = 4.05.

Written Work

1. Add 45.5 + 6.005 + 40.

45.5
6.005
40.
91.505

Keep the decimal points and units of the same order in a column, and add as in whole numbers, placing the decimal point in the sum under the points above.

Find the sums of the following:

2. .1 + .2 + .35 + .365 = ?
3. .02 + .05 + .095 + .056 = ?
4. .05 + .007 + .089 + .11 = ?
5. 1.2 + 3.4 + 4.5 = ?
6. 3.04 + 4.05 + 6.099 = ?
7. .005 + .007 + .009 + .0101 = ?
8. 2.006 + 7.009 + 9.012 = ?
9. .001 + .001 + .0902 = ?

10. $10 + 2.1 + 14.9 + 17.85 = ?$

11. $.9 + .85 + .005 + .25 + .895 = ?$

Add:

12.	13.	14.	15.
1.45	.424	.7	11.111
3.7	8.2	.425	3.06
10.01	6.16	18.54	.635
2.005	19.009	7.011	.000

16.	17.	18.	19.
18.002	.040	89.400	.707
2.056	48.010	75.800	101.101
121.114	.708	761.612	96.086
2.02	89.010	1245.000	27.409

20. Find the sum of 15.38, 9.17, 3.07, and 20.35.

21. A boy picked on Monday, .75 of a bushel of berries; on Tuesday, .875 of a bushel; on Wednesday, 1.125 of a bushel. How many bushels did he pick in the three days?

22. Helen paid \$.25 for a handkerchief, \$2.75 for a pair of shoes, \$.45 for lace, and \$1.49 for a waist. How much did they all cost?

23. A train runs the first hour 19.625 miles; the second hour, 20.5 miles; the third hour, 20.75 miles; the fourth hour, 21.225 miles. How far does it run in the four hours?

24. Find the number of pounds in the following purchases: 1.25 lb. of cheese, 3.5 lb. of sugar, .5 lb. of cloves.

25. The distance from Harrington to Houston is 4.31 miles, thence to Ellendale 11.25 miles, thence to Georgetown 8.37 miles. How far is it from Harrington to Georgetown?

SUBTRACTION OF DECIMALS

Find differences:

1. $.5 - .3 = ?$
2. $.9 - .8 = ?$
3. $15.8 - 11.7 = ?$
4. $4.7 - 3.2 = ?$
5. $.008 - .002 = ?$
6. $.014 - .011 = ?$
7. $.08 + .09 - .12 + .04 + .02 = ?$
8. $.009 + .003 - .007 - .004 = ?$

Written Work

1. From 16.35 subtract 11.76.

16.35
11.76
4.59

Keep the decimal points in a column and subtract as in whole numbers, placing the decimal point in the difference under the points above.

2. 7.	3. 16.	4. 1.101	5. 265.36
1.21	3.046	.796	84.468
6. 25.2	7. 151.003	8. 954.1	9. 36.291
9.18	78.076	258.375	17.456
10. 144.001	11. 300.	12. 86.59	13. 728.3
12.256	261.385	53.594	619.333

14. Warren had $7.50 and spent $3.75. How much had he remaining?

15. The distance between two towns is 9 miles. After I have walked 3.625 miles, how far have I yet to walk?

16. A man having 120 acres of land, sold to one man 28.75 acres, and to another, 35.5 acres. How many acres had he left?

17. If I pay $1.25 for car fare, $.65 for dinner, and $.90 for an umbrella, how much change have I left from a five-dollar bill?

18. The second floor of a house is 18.78 feet above the floor of the cellar, and the first floor is 7.92 feet above it. How far is it from the first floor to the second?

19. Four lots measure in width 123.08 ft. Three of them are respectively 25 ft., 32.72 ft., and 36.9 ft. wide. What is the width of the fourth?

20. A boy having $4.25 spent for skates $1.25, for a cap $.50, and for a hockey stick $.45. How much had he left?

21. A lady having 25.75 pounds of butter sold to one customer 3.25 pounds, to another 8.5 pounds, to another 7.25 pounds, and the balance to a fourth customer. How many pounds did the fourth customer buy?

22. From a ham weighing 18.125 lb. a butcher sold 3.25 lb., 4.50 lb., 2.75 lb., and 2.5 lb. How many pounds had he left?

23. A fisherman brought home four trout weighing respectively 1.25 pounds, .875 pounds, 1.375 pounds, and 1.125 pounds. How much did they all weigh?

24. A farmer cut 40 tons of hay in 1905. He sold 6.85 tons to one man, and 5.55 tons to another. He fed the rest to his stock. How many tons did he feed to his stock?

25. A lady bought 4.75 yards of woolen cloth, 11.625 yards of cotton cloth, and 6.875 yards of silk. How many yards did she buy?

26. A man having $20 spent $4.75 for board, $2.80 for a room, $.88 for laundry, $1.75 for a pair of gloves, and $3.50 for a pair of shoes. How much had he left?

27. A merchant purchased the following: coffee $15.25, sugar $18.35, cakes $11.65, fruit $27.75, canned corn $8.45, canned peaches $12.30, and vegetables $21.90. What was the amount of his bill?

MULTIPLICATION OF DECIMALS

Multiplying a decimal by an integer.

1. $5 \times .3$ means that .3 is taken as an addend 5 times. Thus, $.3 + .3 + .3 + .3 + .3 = 15$ tenths or 1.5.

By multiplication $5 \times .3 = 1.5$.

2. $5 \times .03$ means that .03 is taken as an addend 5 times. Thus, $.03 + .03 + .03 + .03 + .03 = 15$ hundredths or .15.

By multiplication $5 \times .03 = .15$.

3. 5×1.007 means that 1.007 is taken as an addend 5 times. Thus, $1.007 + 1.007 + 1.007 + 1.007 + 1.007 = 5.035$.

By multiplication $5 \times 1.007 = 5.035$.

4. In each problem above, how many decimal places are there in the multiplicand? how many in the product?

Observe that in multiplying a decimal by an integer, the product contains the same number of decimal places as the multiplicand.

Written Work

1. Multiply 5.75 by 6.

$$\begin{array}{r} 5.75 \\ 6 \\ \hline 34.50 \end{array}$$

6×5 hundredths $= 30$ hundredths, or 3 tenths and no hundredths. Write naught in hundredths' place and carry the 3 tenths. 6×7 tenths $= 42$ tenths; 42 tenths + 3 tenths $= 45$ tenths, or 4 units and 5 tenths. Write 5 in tenths' place and carry the 4 units. Place the decimal point. 6×5 units $= 30$ units; 30 units + 4 units $= 34$ units.

Find products:

2. $8 \times .015$	6. $7 \times .97$	10. $86 \times .861$	14. $55 \times .066$
3. $9 \times .005$	7. $12 \times .025$	11. $74 \times .037$	15. $39 \times .467$
4. $8 \times .17$	8. $6 \times .604$	12. $67 \times .92$	16. $48 \times .095$
5. $11 \times .207$	9. $9 \times .054$	13. $44 \times .705$	17. $36 \times .081$

18. How much will 7 arithmetics cost at $.82 apiece?

19. At $.35 apiece, how much will 24 chickens cost?

20. A rod is 16.5 feet. How many feet are there in 9 rods?

21. When a man earns $3.65 per day, how much does he earn in 26 days?

22. A pound of cream cheese costs $.115. How much do 126 pounds cost?

23. If an automobile averages 17.75 miles an hour, how far will it travel in 14 hours?

Multiplying a decimal by a decimal.

1. Multiply 1.5 by 4. When a decimal is multiplied by an integer, what do you observe about the number of decimal places in the product?

2. Multiply .1 by .01; $\frac{1}{10} \times \frac{1}{100} = \frac{1}{1000} = .001$. Multiply 1.5 by .5; $1.5 = \frac{15}{10}$; $\frac{15}{10} \times \frac{5}{10} = \frac{75}{100} = .75$. When a decimal is multiplied by a decimal, what do you observe about the number of decimal places in the product?

Written Work

1. Multiply .75 by .3.

Since there are two decimal places in the multiplicand and 1 in the multiplier, point off 3 decimal places in the product, making the answer .225.

$$\begin{array}{r} .75 \\ .3 \\ \hline .225 \end{array}$$

Test: $.75 = \frac{75}{100}$, and $.3 = \frac{3}{10}$. $\frac{75}{100} \times \frac{3}{10} = \frac{225}{1000}$ or .225, a decimal.

Multiply as in integers, pointing off as many decimal places in the product as there are decimal places in both factors.

Find products:

2. $.8 \times .27$	8. 7.24×8	14. $.025 \times 124$
3. $.5 \times .45$	9. $.011 \times 42$	15. 22.5×4.04
4. $.15 \times .256$	10. $.57 \times .15$	16. $.75 \times .624$
5. 6.5×10	11. $2.03 \times .4$	17. 1.44×5.5
6. 5.7×9.4	12. $.145 \times 48$	18. 2.4×64
7. 3.21×4.5	13. 11.4×150	19. 1.33×44

20. Find the cost of 11.5 yards of cloth at $12\frac{1}{2}$ cents a yard.

21. Mrs. Crane bought 3.5 pounds of steak at \$.16 per pound, 6 pounds of sugar at \$.05 per pound, 4 pounds of coffee at \$.28 per pound, and 3 quarts of cranberries at \$.10 per quart. She gave in payment 3 one-dollar bills. How much change should she receive?

22. How much will 41.25 yards of linoleum cost at \$1.25 per yard?

23. A train between Los Angeles and San Francisco runs 13.5 hours at the rate of 35.7 miles per hour? What is the distance between the two cities?

24. A mail carrier averages 3.25 miles per hour while delivering mail. If he spends 5.25 hours delivering each day, how far does he walk per day?

25. Find the cost of 5.125 yards of silk at \$1.75 a yard. At the same price per yard find the cost of 8.875 yards.

26. At \$2.50 per day, how much will 4 men earn in 6.5 days?

27. A cubic foot of water weighs 62.5 pounds. How much do 12.75 cubic feet of water weigh?

DIVISION OF DECIMALS

Dividing a decimal by an integer.

Written Work

1. Divide .84 by 4 in this way: $4\overline{).84}$ / .21

Divide and test, placing a decimal point in the quotient, before beginning to divide:

2. $6\overline{).66}$	5. $7\overline{).714}$	8. $7\overline{).847}$
3. $3\overline{).96}$	6. $5\overline{).535}$	9. $6\overline{).936}$
4. $8\overline{).808}$	7. $4\overline{).848}$	10. $8\overline{).896}$

11. Explain why adding *naughts* to the right of a decimal does not change its value; thus, .8 = .80, .05 = .050.

It is sometimes necessary to add naughts to the right of the dividend to complete the division.

12. Divide .12 by 5. $5\overline{).12} = 5\overline{).120}$ / .024

Find the quotients and test:

13. $4\overline{).3$	14. $8\overline{).6$	15. $6\overline{).27$	16. $5\overline{).28$
17. .6 ÷ 2	25. .024 ÷ 6	33. .108 ÷ 3	
18. .9 ÷ 3	26. .102 ÷ 3	34. .08 ÷ 2	
19. .12 ÷ 6	27. .039 ÷ 13	35. .125 ÷ 5	
20. .005 ÷ 5	28. .144 ÷ 12	36. .008 ÷ 4	
21. .008 ÷ 4	29. .015 ÷ 3	37. .35 ÷ 7	
22. .27 ÷ 9	30. .063 ÷ 7	38. .077 ÷ 11	
23. .2 ÷ 4	31. .904 ÷ 8	39. .022 ÷ 2	
24. .6 ÷ 8	32. .72 ÷ 10	40. .036 ÷ 6	

Dividing a mixed decimal by an integer.

Written Work

Divide in this way:

1. $6\overline{)6.648}$ 1.108
2. $8\overline{)24.600}$ 3.075
3. $9\overline{)729.83}$ $81.09\frac{2}{3}$

Observe that in dividing a decimal or a mixed decimal by an integer, the dividend is simply separated or *partitioned* into equal parts.

4. Divide 39.25 by 25.

$$\begin{array}{r} 1.57 \\ 25\overline{)39.25} \\ 25 \\ \hline 14.2 \\ 12.5 \\ \hline 1.75 \\ 1.75 \\ \hline \end{array}$$

5. Divide 12.648 by 24.

$$\begin{array}{r} .527 \\ 24\overline{)12.648} \\ 12\,0 \\ \hline 64 \\ 48 \\ \hline 168 \\ 168 \\ \hline \end{array}$$

How many times is 25 contained in 39? in 14.2? in 1.75?

In practice, we simply divide as in the division of integers. Since 24 is larger than 12.648, the quotient must be a decimal.

Place a decimal point in the quotient directly above or below the decimal point in the dividend before beginning to divide; then divide as in the division of integers.

Divide and test:

6. $69.92 \div 23$
7. $29.54 \div 14$
8. $195.2 \div 32$
9. $401.4 \div 18$
10. $3.434 \div 34$
11. $156.4 \div 46$
12. $1.014 \div 26$
13. $5.084 \div 41$
14. $.945 \div 35$
15. $60.32 \div 52$
16. $.968 \div 44$
17. $.828 \div 23$
18. $5.18 \div 37$
19. $.0833 \div 49$
20. $1.566 \div 54$
21. $2.546 \div 67$
22. $8.437 \div 59$
23. $233.32 \div 38$
24. $283.88 \div 47$
25. $6.497 \div 73$
26. $16.150 \div 34$
27. $55.660 \div 92$
28. $5.460 \div 84$
29. $1.6272 \div 18$

30. Divide 12 by 16.

$12 \div 16 = 12.00 \div 16.$

$$\begin{array}{r} .75 \\ 16\overline{)12.00} \\ \underline{112} \\ 80 \\ \underline{80} \end{array}$$

12 is equal to 12.00, which divided by 16 equals .75.

A decimal point must be placed after an integer before naughts are annexed.

Divide:

31. $20 \div 75$	**35.** $44 \div 99$	**39.** 605 by 1210
32. $60 \div 150$	**36.** $110 \div 220$	**40.** 513 by 2052
33. $24 \div 228$	**37.** $340 \div 1700$	**41.** 208 by 1664
34. $30 \div 375$	**38.** $510 \div 1020$	**42.** 111 by 8888

Dividing a decimal by a decimal.

1. $8 \div 4 = 80 \div 40.$
$5 \div 10 = 50 \div 100.$
$6 \div 3 = 600 \div 300.$

2. Compare the quotient of $8 \div 4$ with the quotient of $80 \div 40$. Compare the quotient of $5 \div 10$ with the quotient of $50 \div 100$. Compare the quotient of $6 \div 3$ with the quotient of $600 \div 300$.

$$\frac{.8 \times 10}{.2 \times 10} = \frac{8}{2} \qquad \frac{.08 \times 100}{.02 \times 100} = \frac{8}{2}$$

3. Does multiplying both dividend and divisor *by the same number* affect the value of the quotient?

4. Multiplying by 10 moves the decimal point *how many places to the right?* multiplying by 100?

5. In what short way may a decimal be multiplied by 10? by 100? by 1000, etc.?

6. Multiply .5, .25, .025 each by 10; each by 100; each by 1000.

A decimal is **divided by a decimal** *by moving the decimal point in both dividend and divisor the number of places to the right necessary to make the divisor a whole number, and dividing as in the division of a decimal by an integer.*

Give quotients at sight:

7. $.4 \div .2 = ?$	14. $.06 \div .03 = ?$	21. $.30 \div .10 = ?$
8. $.9 \div .3 = ?$	15. $.05 \div .05 = ?$	22. $.64 \div .16 = ?$
9. $.8 \div .2 = ?$	16. $.21 \div .07 = ?$	23. $.81 \div .09 = ?$
10. $.6 \div .3 = ?$	17. $.12 \div .03 = ?$	24. $.88 \div .08 = ?$
11. $.5 \div .5 = ?$	18. $.24 \div .08 = ?$	25. $.44 \div .11 = ?$
12. $.7 \div .1 = ?$	19. $.42 \div .14 = ?$	26. $.96 \div .12 = ?$
13. $.8 \div .4 = ?$	20. $.15 \div .03 = ?$	27. $.26 \div .13 = ?$

Written Work

1. Divide .025 by .05.

$.05\overline{).025} = 5.\overline{)2.5}$

0.5, *Ans.*

Make the divisor an integer by moving the decimal point two places to the right in both dividend and divisor. Then solve as in dividing a decimal by an integer.

Divide:

2. .325 by .5	11. 2.538 by .27	20. .0095 by .19
3. .756 by .9	12. .5655 by .65	21. 20.25 by .45
4. 6.96 by .8	13. .6150 by .82	22. 6.192 by .72
5. .444 by .6	14. 27.93 by .57	23. 3.360 by .35
6. .888 by .12	15. 52.51 by .59	24. 2.016 by .72
7. 1.445 by .17	16. 6.216 by .74	25. 826.5 by .95
8. 2.695 by .35	17. .6225 by .75	26. 375.2 by .56
9. 3.528 by .42	18. 8.930 by .94	27. 7.728 by .84
10. 3.034 by .37	19. 440.8 by .58	28. 8.025 by .25

Find quotients:

29. $.005 \div 5$
30. $.625 \div .05$
31. $5.55 \div 5$
32. $17.28 \div .12$
33. $3.036 \div .06$
34. $3.728 \div .016$
35. $.864 \div .24$
36. $3.654 \div .21$
37. $10.044 \div .36$
38. $8.007 \div .03$
39. $40.049 \div 1.23$.

$1.23\overline{)40.098} = 123.\overline{)4009.8}$

40. $55.968 \div 1.32$
41. $97.875 \div 2.61$
42. $437.836 \div 5.32$
43. $214.302 \div 3.82$
44. $46.695 \div 1.65$
45. $139.956 \div 3.21$
46. $86.784 \div 2.26$

REVIEW OF DECIMALS

1. At $8.25 per ton, how many tons can be bought for $41.25?

2. From five hundred eighty and sixty-seven ten-thousandths take ninety-six and forty-nine thousandths.

3. John has .75 of $2 and spends .3 of it. How much does he save?

4. If a car conductor earns $1.75 a day, how long will it take him to earn $638.75?

5. If 8.75 tons of coal cost $44.80, how many tons can be bought for $21.76?

6. If I paid $720 for land at $37.50 an acre, how many acres did I purchase?

7. How many tons are there in $27\frac{1}{20}$ tons, 15.7 tons, $9\frac{1}{8}$ tons, and 33.5 tons?

8. To the sum of 14.5 and 9.7 add their difference.

9. What is the value of 7.5 tons of hay at $18.75 per ton?

10. The distance from Pittsburg to San Francisco is 2747.9 miles, and from Pittsburg to Chicago, 507.1 miles. How far is it from Chicago to San Francisco?

When possible, express the numbers in the following decimally:

11. A girl sent 27 pieces to a laundry that charged her seventy-five cents a dozen for washing and ironing them. What was her bill?

12. Mrs. Rorer mixed 15.5 lb. of fat with 2.25 lb. of potash and made soap which she cut into pieces weighing one eighth of a pound each. How many pieces of soap had she?

13. The weight of a diamond before it was cut was 3.875 carats. After it was cut, its weight was 2.50 carats. How much was lost in the cutting?

14. Each of two pillows weighs 4.75 pounds and a bolster weighs $7\frac{1}{8}$ pounds. Find the weight of all.

15. At $\$2\frac{1}{2}$ a ton, how many tons of ice can be bought for $\$3.75$?

16. If I bought 2 gallons of gasoline and used .75 of it to clean a dress, how many quarts did I use?

17. A merchant bought 1200 gas fixtures at $\$.08\frac{1}{3}$ each and sold them at $\$.10$. How much did he make on them?

18. A 24 story city building is 10.75 ft. to a story. How high is the building?

19. Find the value of 17,745 bricks at $\$7.50$ per M.

20. A merchant bought oil at $\$9.43$ per barrel. He sold it at $\$11.48$ a barrel, and gained $\$451$. How many barrels did he have?

21. Divide 5 pounds by .005 of a pound.

22. What is the cost of 350 heads of cabbage at $4.20 a hundred?

23. How many pecks are there in 28.75 bushels?

24. Multiply each of the following by 10: .6, .8, .84, .86, .76.

25. Multiply each of the following by 10: .06, .04, .005, .42, .47, .42.3, 56.7, .478, 8.6, 9.8.

26. Multiply each of the following by 100: .6, .8, .84, .95, .86, .76, .06, .04, .005, 4.23, 56.7, .478, 8.6, 9.8.

27. Multiply each of the following by 1000: .594, 5.94, 59.4, .007, .07, .7, 3.14, 2.5, .0025.

28. Express as hundredths: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{4}{5}$, $\frac{5}{5}$, $\frac{1}{10}$, $\frac{3}{10}$, $\frac{7}{10}$, $\frac{1}{8}$.

29. Divide 4 yards by .004 yards.

30. Compare $9 \div 3$ with $900 \div 300$.

Find the cost of:

31. 24 lb. @ $.125

32. 27 yd. @ $.165

33. 56 bu. @ $.375

34. .875 ft. @ $.48

35. .375 yd. @ $.80

36. .125 T. @ $4

37. 64 bbl. @ $7.50

38. 16 ft. @ $18.75

39. 45 lb. @ $.052

40. 66 gal. @ $.75

41. 2.5 bu. @ $.375

42. 1.25 doz. @ $125

43. What part of 5 is 125?

44. Add the difference between 19.875 and 16.4 to their sum.

45. A merchant withdrew .375 of his bank deposit to invest in property that cost him $6754.50. What was his original bank account?

46. What decimal of $4000 is $800?

47. We pay $.10 for a box containing 100 tags. How much is paid for 1 tag?

48. In January we burned 36,250 feet of gas. At $.25 per thousand, what was the bill?

49. What part of a dollar is $ .005? what part is $ .025?

50. A farmer paid $777.60 for 36 acres of land. How much did he pay per acre?

51. Divide 12 hundredths by 8 thousandths.

52. Mr. Wilson owns 146.875 acres of land. He has 43.8 acres in hay, 35.75 acres in wheat, 26.625 acres in oats, and the rest in pasture. How many acres has he in pasture?

53. Harry picked 3.75 bushels of berries. He sold .5 of a bushel to one neighbor, and 1.25 bushels to another. How much remained?

54. Find the number of pounds in 7 hams, the average weight of each being 8.75 pounds.

55. How much should a man receive who works 15.75 days at $4.40 per day?

56. At $.025 each, how many oranges can I get for $5.40?

57. Compare $30 and $3. What change in the place of the decimal point will give $\frac{1}{10}$ of $30?

58. Add the columns; subtract as indicated; add the remainders in each column:

21.096 − 18.7 =	13.469 − 6.50 =
4.18 − 3.275 =	16.75 − 9.875 =
47.39 − 28.74 =	32.967 − 2.94 =
15.87 − 3.942 =	6.81 − 0.0692 =
26.932 − 25.1 =	7.375 − 1.46 =
− =	− =

59. Add the six quotients:

250 ÷ 125	1 ÷ .05	.25 ÷ .0025
7.50 ÷ 1.25	750 ÷ 1.25	0.75 ÷ 750

PERCENTAGE

In *Common Fractions* we learned that a number may be divided into *any number* of equal parts and any number of these may be taken.

Thus, $\frac{3}{4}$ of 60 means that 60 is divided into 4 equal parts and 3 of these parts are taken.

In *Decimal Fractions* we learned that a number may be divided into 10, 100, 1000, etc., equal parts, and that any number of these parts may be taken.

Thus, .9 of 60 means that 60 is divided into 10 equal parts and that 9 of these equal parts are taken.

We now come to a subject that divides a number into 100 equal parts only. We call this subject **Percentage.**

In common fractions we compute by *halves*, *thirds*, *fourths*, *sixths*, etc.; in decimal fractions we compute by *tenths*, *hundredths*, *thousandths*, etc.; but in percentage we compute by *hundredths* only.

Another name for *hundredths* is *per cent*, usually written "%."

We may now write any number of *hundredths* in three different ways, thus: $\frac{8}{100}$, .08, 8%; $\frac{25}{100}$, .25, 25%.

Percentage is simply an application of decimal fractions.

1. Write the following numbers as per cents:

.05 .03 .15 .20 .25 .40 .06 .75

2. Write the following as decimals:

5% 20% 7% 15% 25% 16% 18% 24% 50% 75%.

3. Show by equivalent decimals that naughts added to the right of a decimal do not affect its value.

4. Write as decimals, and as per cents:

$\frac{1}{2}$ $\frac{3}{4}$ $\frac{1}{20}$ $\frac{1}{25}$ $\frac{1}{10}$ $\frac{2}{5}$ $\frac{4}{5}$ $\frac{3}{8}$

5. What is the difference between .05 of \$100 and 5% of \$100?

6. $5\% = \frac{1}{20}$ of a number; $10\% = \frac{1}{10}$ of a number.

7. 25% of \$100 may be found in two ways: (*a*) $25\% = \frac{1}{4}$; $\frac{1}{4}$ of \$100 = \$25. (*b*) $25\% = .25$; $.25 \times \$100 = \$25$.

Learn the following:

$50\% = \frac{1}{2}$	$16\frac{2}{3}\% = \frac{1}{6}$	$75\% = \frac{3}{4}$
$33\frac{1}{3}\% = \frac{1}{3}$	$12\frac{1}{2}\% = \frac{1}{8}$	$40\% = \frac{2}{5}$
$25\% = \frac{1}{4}$	$10\% = \frac{1}{10}$	$80\% = \frac{4}{5}$
$20\% = \frac{1}{5}$	$5\% = \frac{1}{20}$	$37\frac{1}{2}\% = \frac{3}{8}$

Give per cents at sight:

8. 20% of \$50
9. 25% of \$60
10. 10% of \$40
11. 50% of \$80
12. 40% of \$75
13. 5% of \$40
14. 6% of \$6
15. 75% of \$20
16. $12\frac{1}{2}\%$ of \$72
17. $33\frac{1}{3}\%$ of 30 days.
18. 50% of 60 minutes.
19. 75% of 100 books.
20. 40% of 20 rods.
21. 5% of 40 weeks.
22. $16\frac{2}{3}\%$ of 100 pounds.
23. 10% of 70 bushels.
24. 25% of 24 hours.
25. $12\frac{1}{2}\%$ of 800 bushels.

Written Work

1. Find 28% of 7500 bushels of oats.

7500 bu.
.28
60000
15000
2100.00 bu.

Since percentage is simply so many hundredths of anything, 28% of 7500 bushels equals .28 of 7500 bushels or 2100 bushels.

Find:

2. 27% of $395
3. 14% of $478
4. 24% of $527
5. 6% of $57.40
6. 5% of $90.80
7. 35% of $90.60
8. 40% of $20.50
9. 10% of $2004
10. 5% of $200.60
11. 7% of $500.50
12. 75% of $605
13. 37% of $2005
14. 45% of $6745
15. 80% of $905
16. 98% of $7008

17. Mr. Jordon bought a horse for $175 and sold it for 90% of the cost. For how much did he sell the horse?

18. Raymond has $165 in the savings bank and Bertha has 80% as much. How much more money has Raymond in the bank than Bertha?

19. The distance between two cities is 1080 miles. After 45% of the distance is traveled, how much of the distance remains to be traveled?

20. Mr. Watson earned $1580 in a year, and his son Henry 65% as much. Find the amount Henry earned.

21. The salary of a school teacher last year was $40, and this year her salary was increased 25% of last year's salary. Find her present salary.

22. Paul lives 560 rods from the schoolhouse and David 72 % as far. Find the number of rods David lives from the schoolhouse.

23. Mr. Adams borrows $365 for one year, and pays 6 % for the use of the money for the time. How much money will pay the debt when due?

24. Mr. Brown has loaned $1200 to one party and $1600 to another party. How much does Mr. Brown get each year for the use of the money if each party pays him 5 % of the amount borrowed?

25. Find 5 % of 20; of 40; of 50; of 60; of 80.

26. A newsboy sells $18 worth of papers and gets 40 % for selling. Find his profit.

27. Mary has $24 in the savings bank, and deposits 25 % as much as she has in the bank. Find the amount deposited.

28. A boy borrows $200 to go to school, and pays the lender 5 % for the use of the money for one year. How much does he pay for its use?

29. A boy bought a pony and a cart. The pony cost $80, and the cart 60 % as much as the pony. Find the total cost.

30. A man had 400 sheep. On Monday he sold 25 % of them. On Tuesday he sold 25 % of the remainder. How many sheep had he then?

31. In a spelling test of 30 words, James missed 20 %. How many words did he spell correctly?

32. $\frac{1}{4} = \frac{?}{100} = ?\,\%$. Compare $\frac{1}{4}$ and .20; $\frac{1}{4}$ and 20 %.

33. A man bought a house for $2800. He paid 78 % of the amount cash, and gave his note for the balance. For how much did he give his note?

34. A merchant sold in one year $25,375 worth of goods. His profits were 20 % of the sales. Find his profits.

OPERATIONS WITH UNITED STATES MONEY

10 mills = 1 cent
10 cents = 1 dime
10 dimes = 1 dollar
10 dollars = 1 eagle

1. From the above table tell why United States money is called a decimal system of money.

2. How many cents equal a dollar? What part of a dollar is 1 cent?

3. A cent is what part of a dime? A dime is what part of a dollar? Cents are written as hundredths of a dollar.

4. Read as hundredths of a dollar:

$0.01 $0.02 $0.03 $5.07 $9.09 $8.07.

NOTE. — 1 cent may be written either $0.01 or $.01; 25 cents may be written either $0.25 or $.25, etc. The naught preceding the decimal point does not affect the result, and is sometimes written to show more prominently that cents and not dollars are represented.

5. How many mills equal $1? What part of a dollar is 1 mill? Mills are written as thousandths of a dollar. Thus 1 mill is written $0.001. $0.005 may be read one half cent. Why?

Mills are not coined, but are used for convenience in computations. In final results any part of a cent is usually regarded by the seller as a whole cent. Thus, for a bill amounting to $0.565, we pay $0.57.

6. How does moving a number one place to the right affect its value? one place to the left?

The rules for addition, subtraction, multiplication, and division of decimals apply to United States money since it is a *decimal system.*

Written Work*

1. Find the cost of 27 yards of silk at $\$ 0.87\frac{1}{2}$ a yard.

$\$0.875$
27
6125
1750
$\$23.625$

Study of Problem

1. Why do we change $\frac{1}{2}¢$ to 5 mills?
2. What is the name of the right-hand place in the product?
3. What is the business answer to this problem?

Find the cost of:

2. 4 yd. of lace @ $\$0.37\frac{1}{2}$.
3. 6 bu. of potatoes @ $\$0.62\frac{1}{2}$.
4. 9 doz. eggs @ $\$0.12\frac{1}{2}$.
5. 8 lb. of roast @ $11\frac{1}{2}¢$.
6. 25 bu. apples @ $\$1\frac{1}{8}$.
7. 36 cords of wood @ $\$3\frac{3}{8}$.

Find the amount of:

8. 23 yd. of cloth @ $12\frac{1}{2}¢$.
   14 boxes corn starch @ $8¢$.
   31 lb. of raisins @ $30¢$.
9. 14 cakes of soap @ $2\frac{1}{2}¢$.
   37 boxes macaroni @ $12\frac{1}{2}¢$.
   14 cans of soup @ $10¢$.

10. At $\$1.50$ each, how many readers can be bought with $\$6.00$?

$150¢)600¢(4$ times or readers.
600

When the divisor contains cents, both dividend and divisor may be changed before dividing.

11. At $\$1.25$ each, how many pairs of gloves can be bought for $\$57.50$?

12. A farmer sold potatoes at 2 bushels for $\$1.50$. He received $\$87.50$. How many bushels did he sell?

* For short methods of performing operations of this kind, see p. 175.

13. Walters & Company sold sleds at 65 cents each. They received $31.20. How many dozen sleds did they sell?

14. Find the cost of boarding for 12 weeks at $7.50 a week.

15. A merchant bought $\frac{1}{2}$ dozen pairs of shoes at $2.75 a pair. How much was paid for all?

16. A bookseller sold 56 books at 0.12\frac{1}{2}$ each, 37 books at $0.40 each, and 75 books at $0.25 each. How much money did he receive for all?

17. At $.02$\frac{1}{2}$ each, how much will 1 gross of tablets cost?

18. A man paid $43.75 for carpet at $1.75 a yard. How many yards did he buy?

BILLS

The following is a common form of *bill.*

BOSTON, MASS., *Oct. 10,* 1907.

Mrs. James Brown,
42 Chatham St.

Bought of MORRIS BROS. & Co.,
175 BEACON ST.

TERMS: *Cash.* PHONE 365.

		10 yd. Shirting @ 0.06\frac{1}{2}$	$	65		
		10 " Crash @ 0.06$\frac{1}{4}$		63		
		20 " Calico @ 0.07$\frac{1}{2}$	*1*	*50*	*2*	*78*
		Received payment,				
		Morris Bros. & Co.				
		Per J. B.				

Who sold the goods? Who purchased the goods? When and where was the purchase made? What words show that the bill has been paid?

The words "Received payment, Morris Bros. & Co.," are called the receipt of the bill. Who received the money? When a clerk receives payment for a bill, he always writes the receipt of the firm, per his own name or initials. The receipted bill should be kept by the buyer to show that the bill has been paid.

Every bill should show: (*a*) the *place* and *date* of purchase; (*b*) the *names* of the buyer and the *seller;* (*c*) the *quantity*, the *price* and the *cost* of each item, and the *amount* of the bill.

1. Mrs. James Robinson, on a certain day, buys from Morris Bros. & Co., 12 cans of Acme corn @ $0.12½, 18 pounds sugar for $1.00, 3 pecks potatoes @ $0.25.

Make out the receipted bill.

Make out receipted bills for the following sales, using your father's name as buyer, and the name of your local merchant as seller:

2.	3½ lb. rice	@ $0.08.	3.	12 yd. muslin	@ $0.09.
	10 lb. prunes	@ 0.12½.		10 yd. lace	@ 0.12½.
	2 bags salt	@ 0.10.		2 pair socks	@ 0.35.

4. William Thomas bought of J. A. Crawford & Co., New Castle, Pa., Oct. 10, 1905, 15 lb. butter at 28¢ per lb.; 10 doz. eggs at 24¢ per doz.; 35 lb. lima beans at 11¢ per lb. Make out receipted bill, representing yourself as clerk.

5. Mrs. J. M. Rowe bought of Johnston & Son, Buffalo, N.Y., Dec. 22, 1905, 2 dressed turkeys weighing 12½ lb. and 13 lb. respectively, at 22¢ per lb.; 5 lb. of lamb chops at 19¢ per lb. Clerk, James Brown.

6. W. M. Hays & Son, Baltimore, Md., dealers in general merchandise, sold to Frank N. Clark, Jan. 12, 1905, the following bill of goods:

7¾ lb. butter @ 24¢; 6 lb. cheese @ 11½¢; 14 yd. calico @ 7½¢; 1 can lard, weighing 8¾ lb., @ 12¢.

Supposing that you are the clerk and some neighbor is the buyer, make out receipted bills for the following purchases at your local stores:

7. 8½ lb. lard @ 10¢; 6 cans corned beef @ 15¢; 8½ lb. ham @ 14½¢; 8 lb. sausage @ 12½¢.

8. 3 hassocks @ 98¢; 6 chairs @ $1.25; 12 yd. carpet @ $1.10; 2 rockers @ $2.90; 1 lamp @ $1.65.

9. 1 saw @ 75¢; 3 gas heaters @ $4.90; 3½ doz. screws @ 12¢; 10½ lb. lawn seed @ 20¢; 8 joints stove pipe @ 30¢; 2 elbows @ 40¢.

Another form of bill is commonly used when services have been rendered, as well as material furnished. For example:

Akron, Ohio, June 1, 1907.

Mr. J. R. Burroughs,

To R. W. Jones, Dr.

		To 6 days' Labor @ $1.50	$9	00		
		6 lb. Lawn Seed @ 0.25	1	50		
		8 lb. Nails @ 0.06		48	10	98

Received Payment,
June 18, 1907.
R. W. Jones.

The **creditor** is the person who sells the goods or does the work.

The **debtor** is the person who buys the goods or for whom the work is done.

In the bill on p. 98 Mrs. Brown is *debtor* to Morris Bros., since she owes for the goods purchased, and Morris Bros. are the creditors, since they furnished the goods. In the last bill on p. 100 Mr. Burroughs is the *debtor* for work received, and Mr. Jones is the creditor for work he has done.

1. T. S. Ball owes Dr. S. N. Pool, Lloyd Building, Pittsburg, Pa., for services as follows: Jan. 1, 1907, to 1 call, $2; Jan. 12, 1907, to 1 call, $2; Jan. 14, 1907, office, $1; Jan. 16, 1907, to 1 call, $2. Make out and receipt the bill if paid Feb. 1, 1907.

2. Boydson & Co. owe Charles Frampton, Detroit, Mich., for services as follows:

March 10, 1907, 6 hr. delivering goods	@	$0.20
March 11, 1907, trip to country		2.00
March 12, 1907, " " "		2.00
March 13, 1907, " " "		2.00
March 14, 1907, repairs to wagon		3.75

Write the receipted bill of Boydson & Co., if paid April 1, 1907.

3. James Brown owes Stamm Bros. for labor and material as follows: June 1, 1907, 189 ft. lumber at 8¢ per foot; June 4, 1907, 50 lb. cement at 4¢ per pound; June 8, 1907, 15 days labor at $4.50 per day.

Receipt this bill if paid July 1, 1907.

DENOMINATE NUMBERS

Liquid Measures

A gallon, a quart, and a pint measure should be brought into class. Pupils should measure, and thus learn the relative capacities.

Liquid measures are used in measuring liquids.

2 pints	**= 1 quart (qt.)**
4 quarts	**= 1 gallon (gal.)**
31½ gallons	**= 1 (barrel bbl.)**
63 gallons	**= 1 hogshead (hhd.)**
1 gallon	**= 231 cubic inches**
1 gal. = 4 qt.	**= 8 pt. = 32 gi.**

1. How many pints equal a gallon? a half gallon?
2. How many quarts equal 16 pints? 24 pints? 36 pints?
3. How many gallons equal 28 quarts? 12 qt.? 32 qt.?

Written Work

1. Change 4 gal. 2 qt. 1 pt. to pints.

4 gal. $= 4 \times 8$ pt. $= 32$ pt.
2 qt. $= 2 \times 2$ pt. $= 4$ pt.
1 pt. $= 1$ pt.
4 gal. 2 qt. 1 pt. $= 37$ pt.

Since there are 8 pints in 1 gallon, in 4 gallons there are 4 times 8 pints or 32 pints. Since there are 2 pints in 1 quart, in 2 quarts there are 2 times 2 pints, or 4 pints. 32 pints + 4 pt. + 1 pt. = 37 pt. Hence 4 gal. 2 qt. 1 pt. = 37 pt.

Change:

2. 6 gal. 1 pt. to pints.
3. 8 gal. 1 qt. 1 pt. to pints.
4. 4 gal. 3 qt. to pints.
5. 5 gal. 2 qt. 1 pt. to pints.
6. Change $\frac{7}{8}$ gal. to pints.
7. Mary bought $3\frac{1}{2}$ gallons of cream at 10¢ a pint. How much did it cost her?
8. A grocer sold $6\frac{1}{4}$ gallons of vinegar at 8¢ a pint. How much did he receive for it?
9. Change 127 pt. to gallons.

2)127, no. of pints.
4)63, no. of qt. + 1 pt.
15, no. of gal. + 3 qt.

Since 2 pints = 1 quart, there will be $\frac{1}{2}$ as many quarts as pints; that is, 63 qt. + 1 pt. Since 4 quarts = 1 gallon, there will be $\frac{1}{4}$ as many gallons as quarts; that is, 15 gal. + 3 qt.

NOTE. — The numbers in the operation must be regarded as abstract. Do not say 127 pints ÷ 2 = 63 *quarts* + 1 pint. It is evident that 127 pints ÷ 2 would equal $63\frac{1}{2}$ *pints*.

Change:

10. 375 pt. to quarts.
11. 846 pt. to quarts.
12. 278 pt. to gallons.
13. 675 pt. to gallons.
14. 469 qt. to gallons.
15. 875 qt. to pints.
16. 13 gal. to quarts.
17. 144 gal. to pints.

Dry Measures

Dry measures are used in measuring grain, fruit, roots, and other dry articles. Name five articles sold by the bushel.

2 pints (pt.)	**= 1 quart (qt.)**
8 quarts	**= 1 peck (pk.)**
4 pecks	**= 1 bushel (bu.)**
1 bushel	**= 2150.42 cubic inches**
1 bu. = 4 pk.	**= 32 qt. = 64 pt.**

1. Find, by measuring, how many pecks equal a bushel.
2. How many quarts will fill a peck measure?
3. How many quarts will fill a bushel measure?
4. How many bushels do 16 pecks equal? 64 quarts?

Written Work

Change :

1. 3 bu. 2 pk. 6 qt. to pints.
2. 96 pt. to bushels.
3. 1200 qt. to bushels.
4. 65 pk. to bushels.
5. 8 bu. 3 pk. 1 pt. to pints.
6. 1500 pt. to bushels.

Avoirdupois Weight

The teacher should secure a scale and weights, and have pupils weigh articles of different kinds.

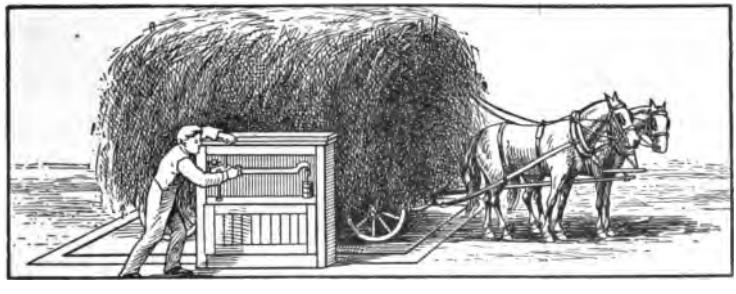
Avoirdupois weight is used in weighing heavy articles; as, groceries, coal, grain, and metals, except gold and silver.

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
20 hundredweight 2000 pounds	= 1 ton (T.)
2240 lb.	= 1 long ton
1 T. = 20 cwt.	= 2000 lb = 32,000 oz.

The **long ton** is used at the United States custom houses, and in wholesale transactions in coal and iron.

The avoirdupois pound contains 7000 grains, and the avoirdupois ounce, 437½ grains.

The unit of avoirdupois weight is the **pound.**



1. Name several kinds of articles sold by the pound; by the ton.

2. How many tons equal 6000 pounds? 24,000 pounds?

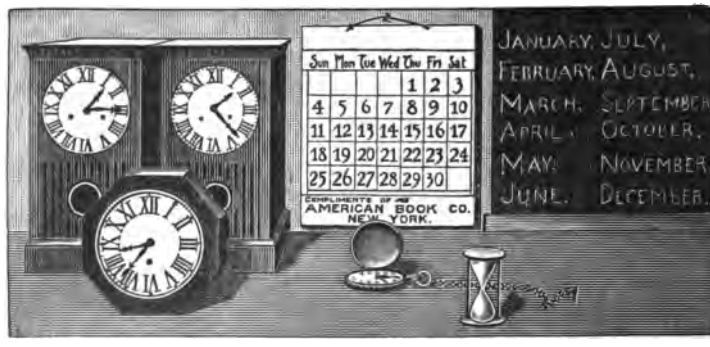
3. Find the cost of 2½ tons of hay at $14 a ton.

Written Work

Change:

1. 6 lb. 5 oz. to ounces.
2. 3 T. 8 cwt. to pounds.
3. 4 cwt. 3 lb. to ounces.
4. $5\frac{1}{2}$ cwt. to ounces.
5. 3600 lb. to cwt.
6. 2 T. 5 lb. to ounces.
7. 544 oz. to pounds.
8. 6000 lb. to tons.
9. 128 oz. to pounds.
10. 810 oz. to lb. and oz.

Time Measures



Time measure is used in measuring time. There are two standard units of time, the **day** and the **year**.

60 seconds (sec.)	**= 1 minute (min.)**
60 minutes	**= 1 hour (hr.)**
24 hours	**= 1 day (da.)**
7 days	**= 1 week (wk.)**
1 year = 12 mo.	**= 365 or 366 days = 52 wk. 1 da. or 52 wk. 2 da.**

1. How many hours is it from midnight on Tuesday to midnight on Wednesday?

2. At what time does the new day begin?

3. How many times in a day does the hour hand pass around the face of the clock?

4. What part of an hour are 10 minutes? 45 min.?

5. What part of a minute are 15 seconds? 30 sec.?

6. What do you mean by A.M.? by P.M.? by M.?

7. Read the time on each of the clocks in the picture.

8. A man leaves home at 6:30 A.M. and returns at 5:45 P.M. How long is he away from home?

9. A train leaves the station at 11:10. It requires 25 minutes to reach the station. At what time must one leave home in order to catch this train?

10. Harry leaves for school at 8:30, and reaches school at 3 minutes before 9 o'clock. How long is he on the way?

11. The morning session of school begins at 9 A.M. and closes at 11:30 A.M. The afternoon session begins at 1 P.M. and closes at 3:45 P.M. How long are both sessions?

12. How many months have 31 days each? 30 days each? 28 days? When has February 29 days?

Written Work

Change:

1. 3 hr. 6 min. to seconds.
2. 144,000 sec. to days.
3. 5 da. 3 hr. to minutes.
4. 108 mo. to days.
5. 12 wk. 6 da. to hours.
6. 336 hr. to weeks.
7. 5760 min. to days.
8. 30 days to seconds.

9. If the school is in session $5\frac{1}{2}$ hours, how many seconds is it in session?

10. At $\$1\frac{1}{2}$ a day, how much will a boy earn in 2 weeks?

11. If a motorman receives 20¢ an hour, what is his pay for 6 days of 9 hours each?

12. A clerk pays $4.50 a week for board. How much will board for 3 months cost him?

13. At the rate of $\$3\frac{3}{4}$ a day, how much can a boy earn during the month of February?

Distance

The teacher should bring into the class a yard stick and a 50-foot measuring line. Each pupil should be provided with a 12-inch ruler, with inches and half inches clearly marked.

Linear measure is used in measuring lines and distances.

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards $16\frac{1}{2}$ feet	= 1 rod (rd.)
320 rods	= 1 mile (mi.)

1 mi. = 320 rd. = 1760 yd. = 5280 ft.

1. **Measure** the length of a book; of a desk; of a table.

2. **Measure** the yard stick with the ruler. How many feet equal a yard? A foot is what part of a yard?

3. With the measuring line mark off $16\frac{1}{2}$ ft., or 1 rod, on the blackboard.

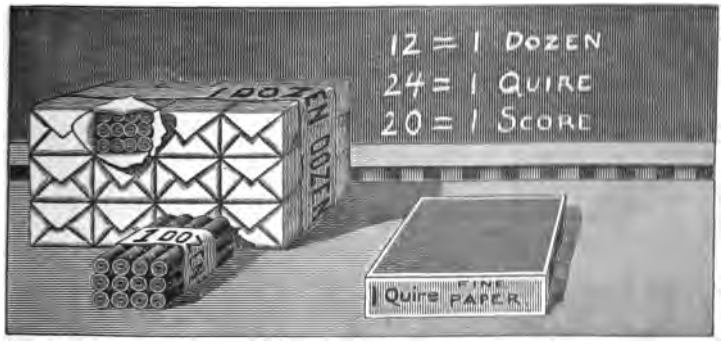
4. How many yards are there in $16\frac{1}{2}$ ft.? in 320 rd.?

Written Work

Change:

1. 33 ft. to inches.
2. 12 yd. to feet.
3. 28 rd. to yards.
4. $\frac{1}{2}$ mi. to inches.
5. 3 yd. 2 ft. 6 in. to inches.
6. 4 rd. 3 yd. 1 ft. to feet.
7. 1728 in. to yards.
8. 198 in. to rods.

Miscellaneous Measures



There are 12 dozen in a gross.

1. How much will 1 gross buttons cost at 15¢ a dozen?

2. A stationer bought 12 quires of paper for $1.20 and sold it at a cent a sheet. How much did he gain?

3. Find the cost of 40 lemons at 15¢ a dozen.

REVIEW

1. A bicycle wheel is 7 feet 4 inches in circumference. How many turns will it make in going 6 miles?

2. How much will $3\frac{1}{2}$ bushels of plums cost at 9¢ a quart?

3. How much fence will be needed to inclose a square field, each side of which is 22 rods?

4. If 9 boxes weigh 27 pounds, how much will 36 boxes weigh? How much will 3 boxes weigh?

5. George has a can of milk containing 10 gallons. If he sells 10 quarts to his first customer, 4 quarts to the second customer, 3 gallons to the third customer, and 2 gallons 1 pint to the fourth, how much has he left in the can?

6. How much are $5\frac{1}{2}$ miles of telegraph wire worth at $0.005 a foot?

7. Our grocer found that 9 hams weighed $82\frac{7}{8}$ pounds. What was the average weight of each ham?

8. How much must be paid for 15 gross of lead pencils at 35¢ a dozen?

9. Walter picked $4\frac{1}{4}$ bushels of blackberries and sold them to a grocer for 6¢ a quart. He took in exchange eggs at 24¢ a dozen. How many dozen did he receive?

10. A butcher sold $30\frac{1}{2}$ pounds of lard at $0.12 a pound, and used the money for flour worth $0.03 per pound. How much flour did he receive?

11. 500 bushels of peaches were packed in baskets, each holding 2 pecks. How many baskets were needed?

12. 20 hundredweight of starch was packed into boxes, each containing 5 pounds. How much was received, if each box was sold for $6\frac{1}{4}$¢?

13. Count the change from a five-dollar bill for 8 pounds of steak at 18¢ a pound, 3 cans of tomatoes at 18¢ a dozen, and 2 gallons of gasoline at 15¢ a gallon.

14. What is my January milk bill, if I use 5 pints every day, at 8¢ a quart?

15. If $\frac{5}{8}$ of a ton of coal costs $3.75, how much will $3\frac{1}{2}$ tons cost?

16. How many pint cans can be filled from 26 gallons of tomato soup?

17. The Bell Telephone Co. charges me $40 a year for 600 calls. How much is that per month? how much for each of the calls?

18. Find the amount of the following sales:

1 dozen boxes of cocoa at 15¢ a box,
8 cans of tomatoes at $1 a dozen,
3 boxes of figs at $3 a dozen,
$11\frac{1}{2}$ pounds turkey at 22¢ a pound.

19. Five girls weigh 75 pounds, 86 pounds, 93 pounds, 69 pounds, and 72 pounds respectively. What is their average weight?

20. There are 20 quires in a ream. At $1.20 a ream, find the cost of 3 reams, 3 quires of paper.

21. How old is a man who is threescore years and ten?

22. The Adams Coal Company sold 8 loads of coal as follows: 2470 lb., 3680 lb., 1974 lb., 2985 lb., 1741 lb., 3164 lb., 3749 lb., and 4278 lb. Find the number of tons, hundredweight, and pounds sold.

PRACTICAL MEASUREMENTS

LENGTHS AND SURFACES

Lines that meet, making a square corner, form a **right angle.**

A figure that has four straight sides and four right angles is called a **rectangle.**

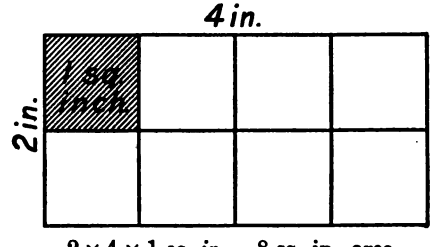
A rectangle having its four sides equal is called a **square.**

Rectangles that are not squares are sometimes called **oblongs.**

144 sq. in. = 1 sq. ft.
9 sq. ft. = 1 sq. yd.

1. What is the **perimeter** or the distance around a surface?

2. What is meant by drawing a surface on a scale of 1 inch to 2 feet? On what scale is this surface drawn? What is the *unit* of measure? How many units are there in the first row of squares? in the second? How many square inches are there in the rectangle, or what is its area?



$2 \times 4 \times 1$ *sq. in.* = 8 sq. in., area.

The **area of a rectangle** *is a number of square units equal to the product of its two dimensions when expressed in like units.*

Thus, if the dimensions of a rectangle are 2 inches and 4 inches, the area is 8 square inches; if the dimensions are 2 feet and 4 feet, the area is 8 square feet.

Written Work

1. If a pane of glass is 10 inches by 12 inches, how many square inches does it contain?

2. How many square feet of glass equal 32 such panes?

3. A garden is 73 feet by 50 feet. How many square feet does it contain?

4. The page of a book is $7\frac{1}{4}$ inches by 5 inches. How many square inches are there in one side of such a page?

5. How many square inches are there in a page of your book?

6. Measure the blackboard in your schoolroom and find how many square feet it contains.

NOTE. — Reduce inches to the fraction of a foot; as, 8 ft. 6 in. $= 8\frac{1}{2}$ ft.

7. How many square inches are there in the surface of your schoolroom door?

8. At $\$1\frac{1}{2}$ per square yard, how much will it cost to cover a floor 12 feet by 15 feet with linoleum?

9. A plate glass window is 9 feet 8 inches wide and 12 feet 3 inches long. How much will such a window cost at $\$0.36$ per square foot?

10. Brussels carpet is $2\frac{1}{4}$ feet wide. How many square feet are there in a yard of it?

11. A room is 16 feet long and 14 feet wide. How much will it cost to paint the ceiling of this room at 12¢ per square foot?

12. Which is the larger, a surface 26 in. long and 5 in. wide or a surface 11 in. long and 12 in. wide?

13. At 15¢ per square foot, how much does a sidewalk 50 feet long and 5 feet wide cost?

14. At 12¢ per square foot, how much will it cost to cement the floor of a cellar 28 ft. 4 in. by 22 ft. 6 in.?

15. At $1.05 per square yard, how much will it cost to pave the street in front of a 50-foot lot, the street being 33 feet wide between the curbs?

16. Compare a square 1 inch on a side with a square 2 inches on a side. Prove your work by drawing the required squares and dividing them into square inches.

17. Compare a rectangle 2 inches by 8 inches with a rectangle 4 inches by 16 inches.

18. Show that a square 4 inches on a side is 16 times as large as a square 1 inch on a side.

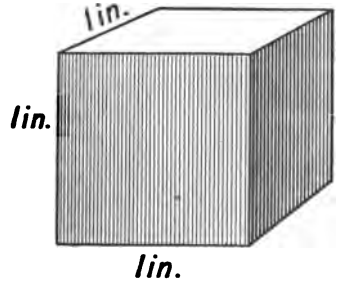
19. Name and draw the figures you have learned in which all the angles are right angles.

20. What other name is sometimes given to a rectangle? Is a square a rectangle? When is a rectangle *not* a square? Draw figure to illustrate.

21. Name the different measures of length.

22. Lines have what dimensions? Surfaces have what dimensions? Illustrate each.

VOLUME



1. What is the length of the figure? the width? the height?

2. How many dimensions has it?

3. How many sides or faces has it?

4. Show that each side is a square.

5. How many square surfaces has it?

A solid with six equal square surfaces is a **cube.**

6. Look at the figure and tell how many edges it has. What is the length of each edge?

A cube whose edge is 1 inch is called a **cubic inch.**

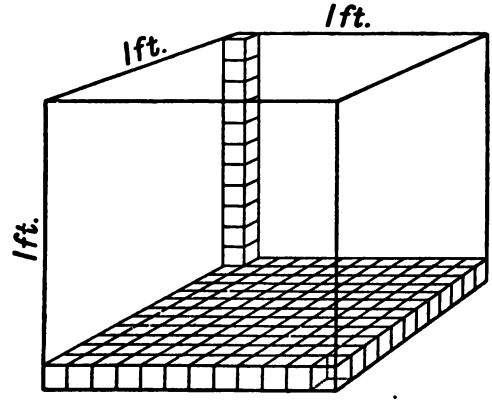
7. Draw on paper or on the blackboard a square foot.

8. Divide each side into 12 equal parts and connect them by straight lines.

9. How many square inches equal a square foot?

10. The base of a 1-inch cube has how many square inches?

11. 144 cubes 1 inch on an edge can be placed on a surface of 1 square foot thus:

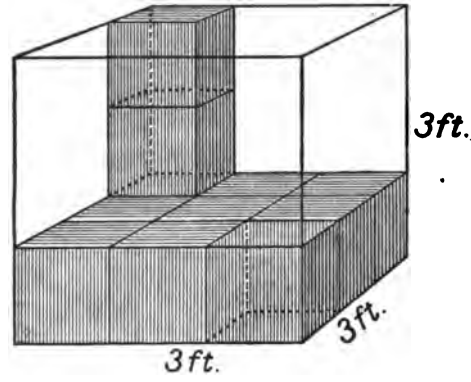


12. What is the height of 12 such layers of cubes? How many cubic inches are there in the first layer? in 12 layers?

13. How many cubic inches can be placed in the cube?

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.)

SURFACES AND VOLUMES



1. How many feet on an edge is the cube in the figure above?

2. How many cubes 1 foot on an edge are there in the first layer?

3. How many cubes 1 foot on an edge are there in the 3 layers?

27 cu. ft. = 1 cubic yard (cu. yd.)
A cart load of earth = 1 cu. yd.

The **contents** *of any body that has six rectangular surfaces is a number of cubical units equal to the product of its three dimensions when expressed in like units.*

4. A schoolroom is 30 ft. wide, 40 ft. long, and 16 ft. high. Find the number of cubic feet of air in it.

5. Find the number of cubic yards of air in the room.

6. A piece of timber is 1 ft. square at one end and 12 ft. long. How many cubic feet are there in it?

7. How many 1-in. cubes are necessary to make a rectangular solid 12 in. long, 8 in. wide, and 4 in. high?

8. A box is 4 ft. long, 2 ft. wide, and 2 ft. high. Find the number of square feet in its six surfaces.

9. Find the number of cubic inches in the box.

10. A bin for grain is 12 ft. long, 8 ft. wide, and 5 ft. deep. Find the number of cubic feet in it.

11. If there are 2150.42 cu. in. in a bushel of wheat, find the number of bushels of wheat the bin will hold.

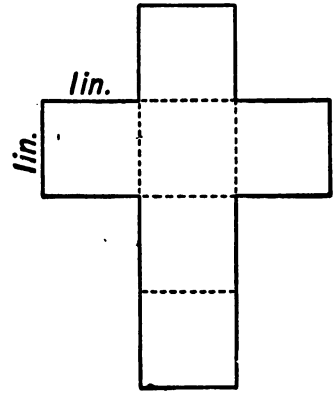
12. A water tank is 8 ft. long, 4 ft. wide, and 3 ft. deep. If a cubic foot of water weighs $62\frac{1}{2}$ lb., find the weight of the water when the tank is full.

13. A stone wall is 40 ft. long, 4 ft. high, and 2 ft. thick. Find the number of cubic feet of stone in it.

14. Measure the dimensions of your schoolroom and find the number of cubic feet of air in it.

15. A rectangular solid is 1 ft. square and 4 ft. long. Find the surface of its six faces.

16. Cut from cardboard a surface as shown in the drawing. Fold along the dotted lines into a box; find the surface of the six faces and the contents of the box in cubic inches.



17. Measure the walls and ceiling of your schoolroom and find the cost of plastering them at 27 cents per square yard.

18. Cut from cardboard a diagram to show a rectangular box 18 in. long, 12 in. wide, and 8 in. deep.

19. Find the number of cubic inches in the box.

20. Measure the surface of your schoolroom floor, and find the cost of oiling it at 12 cents per square yard.

21. What is the difference in cubic feet between 9 cubic feet and a cube 3 feet on an edge?

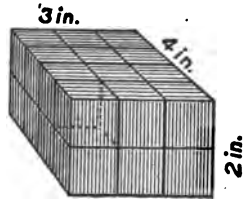
To THE TEACHER. — Get, if possible, 144 cubical blocks 1 inch on an edge.

22. Build a cube 3 blocks on an edge. How many cubic inches are there in the cube?

23. Build a cube 4 blocks on an edge. How many cubic inches are there in the cube?

24. Build a cube 2 blocks on an edge. How many cubic inches are there in the cube?

25. The cube 4 blocks on an edge is how many times the cube 2 blocks on an edge?



26. Build a rectangular solid as shown in the figure. How many cubic inches are there in the solid?

27. Show that the solid in problem 24 has 6 rectangular surfaces. Find the surface of each rectangle.

28. Build a cube 5 inches on an edge. How many more than 100 cubes are needed?

29. Find the cost, at 30 cents a cart load (1 cubic yard), for excavating the ground for a cellar 30 ft. in length, 20 ft. in width, and 4 ft. in depth.

30. A laborer digs a ditch 100 ft. long, 18 in. wide, and $2\frac{1}{2}$ ft. deep in 1 day. Find the number of cart loads of earth removed, and the cost at 30 cents a load.

GENERAL REVIEW

1. A clerk bought a gross of penholders for 80 cents and sold them at 2 cents each. How much did he gain?

2. I paid $3.65 for groceries, $1.75 for dry goods, and 87 cents for sundries. What was the cost of all?

3. Write one hundred two thousand, seven hundred.

4. Find the cost of 30 gross of writing tablets at 37½ cents a dozen.

5. If 11¼ is subtracted from a certain number 1¼ will remain. What is the number?

6. A merchant pays 6½¢ a yard for muslin and sells it for 7¼¢ a yard. What is his gain on 24 yards?

7. I can make 3 aprons in 6 hours. How many aprons can I make in 12 hours?

8. Write in words 4006.015.

9. A man bought ⅓ of a farm containing 300 acres and sold .25 of his share. How many acres had he left?

10. If 8 loads of hay cost $80, what is the cost of 24 loads?

11. Change 12 gallons, 2 quarts, 1 pint, to pints.

12. Robert rides 14.75 miles north on his pony, and his brother James rides 20.25 miles south. How far apart are they at the end of the ride?

13. At Horne's sale a customer bought three remnants of silk, containing respectively 16¼ yd., 12 yd., and 9¾ yd. How many yards of silk did the customer buy?

14. There are 5280 feet in a mile. How many feet is it from Albany to New York, a distance of 143 miles?

15. 6 is $\frac{3}{12}$ of what number?

16. At $50 a month rent, what is the income to the owner on 24 houses for 12 months?

17. At 95 cents per square yard, how much will it cost to cement the floor of a cellar 10 yards long and 6 yards wide?

18. Find the cost of 144 lb. of raisins at $8\frac{3}{4}¢$ a pound.

19. Add 8 bu. 1 pk. 3 qt.
4 bu. 3 pk. 2 qt.
2 bu. 1 pk. 5 qt.

20. Mrs. Black received $2 a yard for broadcloth. She sold 325 yards. How much did she receive?

21. Explain why it is wrong to say $2 times 325 yards.

22. How many feet of picture molding will be required for a room 12 ft. long and 8 ft. wide? At $6\frac{1}{4}¢$ a foot, how much will it cost?

23. Change $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{7}{12}$ to 24ths.

24. Add $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{2}$, and $\frac{7}{12}$.

25. Add by 3's to 54.

26. $32 \times 9\frac{3}{4}$ yards $=$?

27. Change 8 square rods to a fraction of an acre; to a decimal of an acre.

28. Each of 8 boxes holds 5 pounds 4 ounces of meal. How much do all the boxes contain?

29. What is the cost of 8 barrels of vinegar, averaging 41 gallons 3 quarts per barrel, at $18¢$ a gallon?

30. In a warehouse there are 15 pillars 2 ft. by 18 in., and 20 ft. high. How many cubic feet are there in the pillars?

31. How much will it cost, at 27¢ a square yard, to paint a floor that is 18 ft. long and 16 ft. wide?

32. In the first column find the number of yards; in the second, the number of gallons; and in the third, the number of bushels.

Cost	Price per yd.	Cost	Price per gal.	Cost	Price per bu.
\$3.20	$6\frac{1}{4}$¢	\$45.00	$62\frac{1}{2}$¢	\$9.00	75¢
\$6.72	$12\frac{1}{2}$¢	\$36.60	$16\frac{2}{3}$¢	\$15.00	$33\frac{1}{3}$¢
\$8.40	20¢	\$40.50	\$$1\frac{1}{2}$	\$21.75	$37\frac{1}{2}$¢

33. Charles spent at a sale 5 half dollars, 6 quarters, 3 dimes, 4 nickels, and 3 pennies. How much did he spend in all?

34. A motorman worked $8\frac{3}{4}$ hours one day, and $10\frac{1}{2}$ hours another day. At 24¢ an hour, how much did he receive for the two days' work?

35. If 12 T. of hay cost \$144, how much will 24 T. cost?

36. How many pounds of sugar, at \$0.06 a pound, can be exchanged for 9 pounds of butter at \$0.32 a pound, and 12 dozen eggs at \$0.18 a dozen?

37. William works $6\frac{2}{3}$ hours on each of 36 Saturdays. Find the amount earned at 12¢ per hour.

38. If two men can do a piece of work in 8 days, how many men will be required to do the same work in 4 days?

39. If it takes $16\frac{1}{4}$ yards to make a dress, how many yards will it take to make 12 dresses?

40. How much will 128 clocks cost if 8 clocks cost \$104?

41. A lady sold some butter at \$0.32 a pound and with the sum received bought 16 yards of cloth at 90¢ a yard, 4 hats at \$3 each, 4 ties at 24¢ each, and 4 pairs of gloves at 55¢ each. How many pounds of butter did she sell?

42. A father having $105.06 gave $97.08 to his son. If he divided the remaining amount equally between his two daughters, how much did each receive?

43. Find the cost of 81.25 acres of land at $46.75 an acre.

44. At $0.875 per bushel, how much corn can be bought for $52.50?

45. A dressmaker had 40 yd. of cloth. She used $12\frac{1}{4}$ yd. for a dress, $3\frac{3}{4}$ yd. for a coat, and $3\frac{1}{2}$ yd. for a waist. How many yards had she left?

46. Mr. White owned $\frac{3}{4}$ of a farm. He sold $\frac{1}{6}$ of his share at $60 an acre. If the entire farm contained 80 acres, how much money did he receive?

47. At $12\frac{1}{2}$ cents a dozen, how much will $\frac{3}{4}$ of 16 dozen buttons cost?

48. If $\frac{2}{3}$ of a dozen oranges cost 20 cents, how much will $5\frac{1}{3}$ dozen cost?

49. A company of four men leased a tract of West Virginia land and drilled an oil well. It flowed for a time at the rate of 100 barrels a day. If oil was selling at $1.68 a barrel, how much did they realize per day?

50. I paid $6.30 for ribbon. If I paid 60¢ for $\frac{2}{3}$ of a yard, how many yards did I buy?

51. A man sells 27.5 casks of wine, each containing 31.25 gallons, from a stock of 1000 gallons. How much remains?

52. A bought a horse for $100, a cow for 45\frac{1}{2}$, and a wagon for 78\frac{3}{4}$. What was the cost of all?

53. $65 is the value of $\frac{5}{6}$ of an acre of land. At this rate how many acres of land can be bought for $6396?

54. Change 27000 pounds to tons.

55. One side of a square is 18 ft. 8 in. What is the distance around it?

56. If I spend $\frac{3}{4}$ for an almanac, $\frac{1}{2}$ for a book, and $\frac{1}{4}$ for paper, how much do I spend in all?

57. Lucy has $\frac{3}{4}$, and Mary $\frac{1}{2}$. Which has the more, and how much more?

58. Charles earned $17\frac{1}{2}$, John $20\frac{1}{4}$, and William $25\frac{1}{2}$. How much more must they earn to have $75 in all?

59. Perform the operations indicated: $\frac{5 \times 8.75}{1\frac{1}{4} \times 5\frac{1}{2}}$.

60. On Jan. 1, 1901, Mr. Bethrow bought of T. C. Jenkins & Co., the following:

25 bbl. of flour,	@	$ 5.25
20 bbl. of sugar,	@	14.50
29 hams,	@	1.85
112 lb. of bacon,	@	0.10

Find total, and receipt the bill, acting as clerk for T. C. Jenkins & Co. yourself.

61. How many pecks are there in $17\frac{1}{2}$ bushels?

62. $(.5 + .75) \div .00125 = ?$

63. In an orchard there are 144 trees; 18 are cherry, 36 are apple, and $\frac{1}{6}$ of the remainder are peach trees. How many are peach trees?

64. How many pecks are there in 25 bu. 3 pk.?

65. Find the cost of $21\frac{3}{4}$ bushels wheat at 80¢ a bushel.

66. 1 ton of coal will cost how many times as much as 5 hundredweight? as 600 pounds? as 60 pounds?

67. A haystack contains 9000 pounds of hay. How much is it worth at $15 a ton?

68. A grocer bought 5 barrels of cider at $6.50 a barrel, and sold it at $0.10 a quart. How much did he gain?

69. A farmer exchanged 45 pounds of butter, at 30 cents a pound, for sugar, at 5 cents a pound. How many pounds of sugar did he receive?

70. How many cubic feet of air are there in a room 12 ft. long, $10\frac{1}{2}$ ft. wide, and 9 ft. high?

71. What is the cost of 48 gal. oil at 5 cents a quart?

72. For $\$20\frac{1}{4}$, how many barrels, at $\$2\frac{1}{4}$ a barrel, can be bought?

73. A boy lost $\frac{1}{4}$ of his marbles one day, $\frac{1}{5}$ of them the next day, and had 44 marbles left. How many had he at first?

74. John had 120 marbles. He gave Louis $\frac{2}{5}$ of them, and Henry $\frac{1}{3}$ of the remainder. How many had he remaining?

75. If a boy pays $\$2.50$ a hundred for papers, and sells them for 5 cents apiece, how much does he gain on 300 papers?

76. A field contains 25 rows of corn. If each row yields 5 bu. 3 pk., how much corn will the field yield?

77. What will be the cost of painting the ceiling and the floor of a kitchen 14 ft. long and 12 ft. wide, at $\$0.10$ per square yard?

78. If a boy's wages are $\$1\frac{1}{4}$ per day, how much will he earn in 14 days?

79. How many loads of earth will be removed in excavating a cellar 24 ft. long, 18 ft. wide, and 6 ft. deep?

80. Add one and one tenth, one and one thousandth, one hundred and one thousandth, ten and five hundredths, and six tenths.

81. Frank worked $2\frac{1}{2}$ hours each day and 12 hours on Saturday. Find his earnings for 10 weeks at $12¢$ an hour.

82. What is the relation of 2 pecks to 4 bushels? Express the relation both in common and decimal fractions.

83. Divide 24 by .24.

84. 12.50 is $\frac{1}{8}$ of what number?

85. From ten tenths take one ten-thousandth.

86. Why is the expression \$ $6\frac{1}{2}$ times 425 chairs wrong? Why is \$ $6\frac{1}{2}$ times 425 wrong?

87. If $12\frac{1}{2}\%$ of a flock of 960 sheep are sold, how many are left?

88. A market gardener bought 20 yards of carpet at \$1.25 a yard and paid for it in potatoes at \$0.50 a bushel. How many bushels were required?

89. If a bushel of wheat weighs 60 pounds, how much will 3 pecks weigh?

90. If 8 lb. tea cost \$4.80, how much will 24 lb. cost?

91. If huckleberries are selling at 10¢ a quart, how much will be received for 4 quarts 1 pint?

92. A dairyman buys milk at \$0.24 a gallon and retails it at a profit of $33\frac{1}{3}\%$. How much does he gain on one gallon? on 50 gallons?

93. After selling $25\frac{1}{2}$ yd. muslin from a web, there remain $15\frac{3}{4}$ yd. How many yards were in the web at first?

94. At a distance of 9 ft. apart, how many fence posts will be required to fence a lot 150 ft. long and 75 ft. wide?

95. $(2.5 \times 2.5) \div 6.25 = ?$

96. At \$4.50 a day, what amount will a man receive for working 4 weeks of 6 days each?

97. A property holder bought a brick house for \$4524 and sold it at a gain of 75%. For what sum did he sell it?

PART II — SIXTH YEAR

REVIEW OF NOTATION AND NUMERATION

The figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 are called Arabic numerals, and the method of representing numbers by these figures is called the **Arabic notation.**

1. In 555 the first place at the right is called —— place; the second place is called —— place; the third place is called —— place.

2. The 5 tens are how many times the 5 ones? The 5 hundreds are how many times the 5 tens? Upon what does the value of a figure in any number depend?

3. A cent is what part of a dime? A dime is what part of a dollar?

In our system of notation and of money, 10 *ones of any place make* 1 *ten* of the next higher place.

4. Compare 20, 200, 2000. How is any integer affected by adding one naught to the right? two naughts? three naughts?

5. How is any integer affected by removing one naught from the right? two naughts? three naughts?

Separate into periods, enumerate, and read:

6. 600247015	9. 247100503	12. 270600005
7. 920369400	10. 301000004	13. 605702609
8. 500010369	11. 630630630	14. 807000001

Never use *and* in reading whole numbers.

15. In example 11, how many times greater is the period to the left than the right-hand period? than the middle period?

16. The expenses of our government in one year were $1785687098. Separate this number into periods and read it.

17. Write four other numbers expressing billions.

Write:

18. Ten thousand ten hundred two.

19. Six hundred millions six thousand six hundred.

20. Six billion sixty-six million ten thousand five.

21. Five hundred five billion ninety-eight thousand four hundred four.

22. Eight hundred million eight.

23. Six million five.

REVIEW OF ADDITION

1. Define addition; addends; plus; sum.

2. What must be observed in writing numbers to be added?

3. Add the numbers in examples 18–23 above.

Add:

4. 4, 2, 6, 8
5. 3, 7, 1, 9
6. 5, 4, 6, 5
7. 2, 3, 8, 7
8. 5, 4, 1, 3
9. 6, 2, 4, 8
10. 30, 60, 10, 5
11. 22, 30, 20, 19
12. 25, 14, 25, 30
13. 18, 32, 30, 20
14. 50, 25, 25, 60
15. 40, 15, 60, 25

Give sums at sight:

16. 40, 60, 50	21. 42, 18, 7	26. 21, 14, 15
17. 65, 35, 40	22. 16, 32, 22	27. 48, 22, 69
18. 92, 28, 31	23. 72, 18, 35	28. 50, 65, 15
19. 86, 14, 25	24. 16, 24, 39	29. 25, 15, 23
20. 40, 15, 65	25. 87, 13, 25	30. 37, 62, 38

A large number of problems may be made from the following numbers. Add horizontally or vertically to a certain point; subtract the numbers in one column from those in a column to the right; or multiply the numbers in the last column by those in the first column, etc.

In adding, notice convenient combinations.

	a	*b*	*c*	*d*	*e*
31.	594	697	913	1043	8697
32.	273	598	870	1195	7963
33.	634	736	837	1375	8674
34.	175	214	415	2867	3695
35.	439	567	638	4298	9369
36.	275	324	475	5149	6937
37.	684	768	867	6238	7245
38.	273	436	537	2628	8734
39.	619	875	946	4153	6928
40.	875	987	987	1697	7961
41.	267	349	693	4278	8639
42.	286	483	679	4284	5698
43.	394	526	638	6953	7293
44.	899	743	698	8907	5032

REVIEW OF SUBTRACTION

1. Define subtraction, minuend, subtrahend, remainder, minus, difference.

2. How may we test subtraction?

Give differences at sight:

3. $43-28$	6. $86-43$	9. $31-14$	12. $86-79$
4. $65-32$	7. $71-18$	10. $26-19$	13. $96-93$
5. $94-77$	8. $69-27$	11. $37-24$	14. $18-11$

15. When the minuend and the difference are given, how may the subtrahend be found?

16. When the difference and the subtrahend are given, how may the minuend be found?

Take each number below from 1000:

17. 225	21. 216	25. 725	29. 715	33. 375
18. 314	22. 500	26. 946	30. 800	34. 814
19. 625	23. 499	27. 328	31. 125	35. 731
20. 374	24. 795	28. 613	32. 625	36. 656

37. Take each of the numbers in examples 17–36 from 10000.

Subtract in 3 minutes:

38. 607008	41. 180260	44. 800647900
448789	98775	98749897
39. 756008	42. 100907	45. 870009809
398497	49899	698058948
40. 640006	43. 230900	46. 906700983
290809	97897	798897497

REVIEW OF MULTIPLICATION

1. Show that multiplication is a *short* method of addition.

2. Define multiplication, multiplier, multiplicand, product, factors, an abstract number, a concrete number.

3. What kind of a number must the multiplier always be?

4. What does the multiplication sign show? Which term in multiplication is usually written before it?

5. At \$2 a bushel, how much will 125 bushels of peaches cost? Which number is the multiplicand? Which one is the multiplier? $125 \times \$2 = ?$ How many times has the multiplicand \$2 been taken? What is the product? Why is the product a concrete number?

6. Compare the product of 8×6 with the product of 6×8; the product of 2×14 with the product of 14×2. *Either factor may be regarded as the multiplier.*

The multiplicand may be either concrete or abstract. When it is concrete, the product will have the same name as the multiplicand. The multiplier is *always* abstract. Hence, when a concrete multiplicand is for convenience used as multiplier, it must be *regarded as abstract.*

Name two factors of:

7. 81; 125; 75; 64; 110; 108; 39; 72; 96; 80.

8. Multiply by 10, by 100, by 1000: (Do not use pencil.) 4; 8; 12; 15; 18; 25; 30; 35; 40; 50; 75.

9. State how the addition of one naught, two naughts, three naughts, etc., to the right of a number affects its value.

State products:

10. 40×20	13. 50×20	16. 64×40
11. 20×20	14. 18×30	17. 70×28
12. 30×15	15. 30×70	18. 40×70

Find the products:

19. 309×785	23. $386 \times \$41.72$	27. $693 \times \$37.14$
20. 597×900	24. $648 \times \$65.39$	28. $245 \times \$64.59$
21. 987×409	25. $269 \times \$73.36$	29. $369 \times \$83.96$
22. 500×690	26. $845 \times \$69.47$	30. $248 \times \$39.81$

31. Find the amount of the following bills:

16½ lb. butter at 18¢ a pound.
12¾ doz. lemons at 20¢ a dozen.
6 cans corn at 2 for 25¢.
12 lb. roast at 12¢ a pound.

32. 18 bu. potatoes at $0.65 a bushel.
25 bu. tomatoes at $0.55 a bushel.
3 boxes peaches at $1.75 a box.
15 qt. berries at 10¢ a quart.

33. Make a bill for 6 articles bought at a dry goods store.

34. Take the number 125, multiply it by 5, then this product by 5, and so on. Write only the products, not the multipliers. Continue for two minutes. How many successive products have been written?

REVIEW OF DIVISION

1. Define division, dividend, divisor, quotient, remainder.

2. What is the sign of division? Division is indicated in three ways; thus, $15 \div 5$, $\frac{15}{5}$, and $5)15$.

3. In problem 2, which number is the dividend? which is the divisor?

If the dividend and divisor are concrete, they must have the same name. The quotient is then abstract. Thus, $7 (divisor) is contained in $21 (dividend) 3 times (quotient).

When the divisor is abstract and the dividend concrete, the quotient has the same name as the dividend. Thus: $21 ÷ 7 = $3, or ⅐ of $21 = $3.

When we consider that $7 is contained 3 times in $21, the problem differs from the separation of $21 into 7 equal *parts*. The latter kind of division is called **partition.**

4. Divide the following first by 10; then by 100; then by 1000: 5000; 35000; 42000; 63,000; 78,000. How does the removal of one naught from the right of a number affect its value? the removal of two naughts? three naughts? etc.

5. Find ⅛ of 120; 160; 200; 400; 480; 960.

6. How do you prove division? Illustrate.

Give the quotients at sight:

7. 160 ÷ 4	12. 720 ÷ 24	17. 750 ÷ 15	22. 880 ÷ 4
8. 280 ÷ 7	13. 900 ÷ 15	18. 400 ÷ 4	23. 1200 ÷ 20
9. 960 ÷ 8	14. 500 ÷ 2	19. 360 ÷ 6	24. 7200 ÷ 60
10. 1080 ÷ 12	15. 900 ÷ 45	20. 360 ÷ 5	25. 9000 ÷ 60
11. 900 ÷ 5	16. 100 ÷ 25	21. 900 ÷ 3	26. 3600 ÷ 30

27. Divide the number 4,294,967,296 by 4; divide the quotient by 4, and continue to write successive quotients by 4 as divisor for two minutes without writing divisors. Change the dividend and the divisor until rapidity and accuracy have been attained.

Find quotients and test:

28. 23021 ÷ 49	34. 97009 ÷ 69	40. 367047 ÷ 734
29. 72012 ÷ 85	35. 78007 ÷ 89	41. 908006 ÷ 906
30. 70057 ÷ 91	36. 52475 ÷ 62	42. 780980 ÷ 694
31. 77818 ÷ 67	37. 70601 ÷ 96	43. 600809 ÷ 710
32. 75349 ÷ 78	38. 70075 ÷ 56	44. 740096 ÷ 897
33. 77445 ÷ 86	39. 42882 ÷ 74	45. 873401 ÷ 732

46. Find the average of 465 bu., 373 bu., 914 bu., and 762 bu.

47. Find the average of 613 ft., 724 ft., 484 ft., and 465 ft.

48. How many hats, at $1.25 each, can be purchased for $57.50?

49. $108.75 was received from a sale of china plates at $0.75 each. Find the number of plates that were sold.

50. A Reading Circle purchased copies of *Longfellow's Poems* at $1.35 each. The entire amount paid was $25.65. How many copies were bought?

51. Find the number of dozen eggs at $12\frac{1}{2}$ cents a dozen that can be bought for $32.25.

52. Bicycles were sold at $22.50 each. A check for $1012.50 was given in settlement. Find the number bought.

REVIEW OF DENOMINATE NUMBERS

1. Change 12 gallons 1 pint to pints.
2. How many inches are there in 14 feet 10 inches?
3. How many feet and inches are there in 1020 inches?
4. Compare in length 6 inches and 2 yards.
5. Compare $2\frac{1}{2}$ yards and 2 rods.
6. How many yards are there in 2700 inches?
7. Change 3 T. 6 cwt. 12 lb. to pounds.
8. How many seconds are there in $5\frac{1}{2}$ hours?
9. How many 8-ounce packages of soda can be put up from 1 ton 300 pounds of soda?
10. A field is 80 rods long and 320 feet wide. How many yards is it around the field?

11. James is in school $5\frac{1}{2}$ hours each day for 180 days. How many days of 24 hours each would this equal?

12. How many miles does William walk in going to and coming from school, in a term of 140 days, if he lives 600 yards from school and attends every day?

13. Mr. Hosack feeds his horse 4 quarts of oats 3 times a day. How many bushels of oats does he feed the horse during November, December, and January?

MISCELLANEOUS REVIEW

1. Compare 64 and 8; 21 and 7; 90 and 15.
2. 3 square feet is what part of a square yard?
3. Compare 36 inches and 6 inches.

Compare:

4. 4 rods and 3 feet.
5. A yard and a rod.
6. A cube 1 foot on an edge and one 6 inches on an edge.
7. A pint and a gallon.
8. 3 pecks and 3 bushels.
9. What part of a square yard are 2 square feet?
10. What part of a bushel are 8 quarts?
11. What part of a rod are $8\frac{1}{4}$ feet?
12. What part of a mile are 528 feet?
13. What part of 2 bushels are 2 pecks?
14. What part of a gross are 2 dozen?

Make problems and solve them:

15. A floor is 16 feet long and 12 feet wide.
16. A floor is 8 yards long and 6 yards wide.

17. A grocer bought 8 bushels 2 pecks of onions.

18. How many pint bottles can be filled from ——?

19. A room 12 feet long, 10 feet wide, and 9 feet high is to be painted.

20. A square contains 36 square feet.

21. Find the number of cubic inches in a rectangular solid —— × —— × ——.

22. A wall is 25 ft. long.

23. $\frac{2}{3}$ of the cost of a house is $1200.

Announce the products:

24. 60×70	27. 50×70	30. 125×40
25. 80×90	28. 200×60	31. 150×50
26. 90×60	29. 250×40	32. 160×60

Announce the quotients:

33. $840 \div 120$	35. $240 \div 120$	37. $7200 \div 120$
34. $960 \div 80$	36. $6400 \div 80$	38. $8100 \div 90$

39. What two factors will produce 60? 72? 95?

40. What two equal factors will produce 9? 16? 144?

41. Square 60; 90; 25; 15; 40.

42. What two equal factors will produce 3600? 8100?

Find the value of:

43. $(60 + 40) \div 10$

44. $(3 \times 80) \div 6$

45. $8 + \overline{(4 \times 3) \div 5}$

46. $\frac{24}{8} + \frac{60}{4} + \frac{75}{5}$

47. $(4 \times 6) + (15 \div 5)$

48. $\dfrac{6 \times 8 \times 10 \times 100}{3 \times 5 \times 25}$

49. How does annexing a naught to a number affect its value?

50. How does removing a naught from the right of a number affect its value?

FACTORING

1. What are the factors of 4? of 6? of 8?

The **factors** of a number are the integers that, multiplied together, will produce that number. They are **exact divisors** of the number since they will divide it an integral number of times.

Thus the factors of 4 are 2 and 2, since $2 \times 2 = 4$; the factors of 6 are 2 and 3, since $2 \times 3 = 6$; the factors of 8 are 4 and 2, since $4 \times 2 = 8$, or 2, 2, and 2, since $2 \times 2 \times 2 = 8$.

2. What two factors will produce 9? 15? 25? 32?

3. If 3 is one of the factors of 36, what is the other factor? if 4 is one of the factors?

4. Name two factors that will produce 16; 24; 28; 38; 46; 52.

5. What three factors will produce 8? 12? 20? 42?

6. What number will divide 13, giving a whole number as quotient? Observe that 13 has no factors except itself and 1.

A **prime number** is an integer that has no other factors except itself and 1. All other integers are called **composite numbers.**

Thus, $5 = 5 \times 1$; $7 = 7 \times 1$. 5 and 7 are prime numbers. 2, 4, 6, 8, 9, etc., are composite numbers.

7. What numbers between 0 and 30 are prime numbers? between 30 and 60?

Prime factors are factors that are prime numbers.

Thus, 3 and 7 are the prime factors of 21.

TESTS OF DIVISIBILITY

1. Divide 12, 14, 24, 26, 36, 48, and 50 each by 2. What is the ones' figure, or **digit**, in each number? Divide other numbers ending in 2, 4, 6, 8, or 0 by 2.

A number is divisible by 2, *if the ones' figure is* 2, 4, 6, 8, *or* 0.

2. Divide 5, 15, 20, 65, 155, 180, and 240 each by 5. What is the ones' figure, or digit, in each number? Divide other numbers ending in 5 or 0 by 5.

A number is divisible by 5, *if the ones' figure is* 5 *or* 0.

3. Divide 3, 33, 39, 69, 129, 339, and 369 each by 3. Is the sum of the figures, or digits, in each of these numbers divisible by 3? Divide by 3, other numbers the sum of whose digits is divisible by 3.

A number is divisible by 3, *if the sum of the digits is divisible by* 3.

4. Divide 9, 18, 36, 81, 189, and 918 each by 9. Is the sum of the digits in each of these numbers divisible by 9? Divide by 9 other numbers the sum of whose digits is divisible by 9.

A number is divisible by 9, *if the sum of the digits is divisible by* 9.

5. State which of the following numbers are divisible by 2; by 5; by 3; by 9:

6. 10	9. 135	12. 400	15. 1350	18. 2500
7. 54	10. 270	13. 600	16. 1676	19. 3240
8. 90	11. 324	14. 800	17. 1732	20. 9600

21. Which of the foregoing numbers are divisible both by 2 and by 5? by 2, 3, and 5? by 2, 3, and 9? by 2, 3, 5, and 9?

PRIME FACTORS

Find by inspection the prime factors of:

1. 12	3. 27	5. 35	7. 45	9. 54
2. 21	4. 32	6. 40	8. 52	10. 60

Written Work

1. Find the prime factors of 120.

$$\begin{array}{r|l} 2 & 120 \\ 2 & 60 \\ 2 & 30 \\ 3 & 15 \\ & 5 \end{array}$$

Divide 120 by the prime factor 2, which gives the quotient 60. Divide 60 and the succeeding quotients by the smallest prime factors that will divide each. The last quotient, 5, is a prime number. The prime factors of 120 are 2, 2, 2, 3, 5; that is, $120 = 2 \times 2 \times 2 \times 3 \times 5$.

Every composite number is equal to the product of all its prime factors.

2. Find the prime factors of 700.

$100)\overline{700}$
7

$100 = 10 \times 10 = 2 \times 5 \times 2 \times 5$

$\therefore 700 = 2 \times 5 \times 2 \times 5 \times 7$

We see at a glance that $700 = 100 \times 7$, or $10 \times 10 \times 7$. Since the factors of 10 are 2 and 5, the factors of 700 are 2, 5, 2, 5, and 7.

Find the prime factors of:

3. 96	11. 240	19. 600	27. 1849
4. 100	12. 360	20. 900	28. 4032
5. 108	13. 432	21. 1296	29. 6935
6. 120	14. 576	22. 1576	30. 6300
7. 125	15. 720	23. 1728	31. 9000
8. 160	16. 960	24. 2016	32. 64000
9. 180	17. 300	25. 2592	33. 99999
10. 210	18. 400	26. 2310	34. 21504

CANCELLATION

Cancellation is the process of shortening operations in division by striking out equal factors from dividend and divisor.

$$48 \div 16 = 3.$$

We may divide the dividend 48 into the factors 8 and 6, and the divisor 16 into the factors 8 and 2. Therefore, we may write this example in the following way:

$$(8 \times 6) \div (8 \times 2) = 3.$$

If we strike out the equal factor 8 in both dividend and divisor, the problem reads:

$$6 \div 2 = 3.$$

Striking out equal factors from both dividend and divisor does not change the value of the quotient.

Written Work

1. Divide 10×25 by 5×10.

$$\frac{5}{\cancel{10} \times \cancel{25}} = 5$$
$$\cancel{5} \times \cancel{10}$$

Strike out from both dividend and divisor the factor 10. In a similar manner strike out from both dividend and divisor the factor 5, leaving 5 in the dividend and 1 (which need not be written) in the divisor. The result is 5.

Divide as indicated:

2. $\frac{20 \times 35}{4 \times 7} = ?$

3. $\frac{15 \times 50 \times 5}{3 \times 10 \times 25} = ?$

4. $\frac{18 \times 16 \times 9}{6 \times 9 \times 4} = ?$

5. $\frac{27 \times 36 \times 60}{9 \times 18 \times 10} = ?$

6. $\frac{22 \times 30 \times 40}{6 \times 8 \times 11} = ?$

7. Divide 54×128 by 9×4.

8. Divide 720×120 by 60×80.

9. Divide 630×110 by 55×90.

10. Divide 625×255 by 25×85.

11. Divide 1000×147 by 125×21.

GREATEST COMMON DIVISOR

1. Name a number that will exactly divide 8 and 12; 10 and 15; 12 and 16.

A **common divisor** (c. d.) of two or more numbers is a number that exactly divides each of them; thus, 4 is a common divisor of 16 and 24.

The **greatest common divisor** (g. c. d.) of two or more numbers is the *greatest* number that will exactly divide each of them; thus, 8 is the g. c. d. of 16 and 24.

2. Name the g. c. d. of 12 and 16; of 20 and 30; of 18 and 27; of 10 and 15; of 22 and 33; of 63 and 81.

The **greatest common divisor** *of two or more integral numbers is the product of all their common prime factors.*

Written Work

1. Find the greatest common divisor of 24, 36, and 48.

2)	24	36	48
2)	12	18	24
3)	6	9	12
	2	3	4

Divide all the numbers by a common prime factor. In the same way divide the quotients until they are prime to each other. The divisors 2, 2, and 3 are *all the common prime factors.* Hence the g. c. d. of 24, 36, and 48 is $2 \times 2 \times 3$, or 12.

2. Find the greatest common divisor of 165 and 210.

3)	165	210
5)	55	70
	11	14

Dividing each number by 3 and the quotients by 5, we find that the only prime factors common to both are 3 and 5. Hence their product, 15, is the greatest common divisor of 165 and 210.

Find the greatest common divisor of:

3. 72 and 108
4. 36 and 90
5. 391 and 697
6. 135 and 270
7. 408 and 544
8. 190 and 570

NOTE. — The chief application of greatest common divisor is in reducing fractions to their lowest terms. See pp. 148, 149.

LEAST COMMON MULTIPLE

1. Name a number that contains 3 and 4 a whole number of times; 4 and 5.

A **common multiple** of two or more numbers is a number that can be exactly divided by each of them; thus, 24 is a common multiple of 3 and 4.

2. Name the *least* number that can be exactly divided by 3 and 4; by 6 and 8; by 5 and 6.

The **least common multiple** (l. c. m.) of two or more numbers is the least number that can be exactly divided by each of them; thus, 12 is the l. c. m. of 3 and 4.

3. Name the l.c.m. of 4 and 5; of 6 and 8; of 6 and 9.

The **least common multiple** *of two or more numbers is the product of all their prime factors each used as often as it occurs in any one of the numbers.*

Written Work

1. Find the l. c. m. of 16 and 24.

$16 = 2 \times 2 \times 2 \times 2$
$24 = 2 \times 2 \times 2 \times 3$

2 occurs 4 times as a factor in 16. It must, therefore, be used 4 times in the l. c. m. 3 occurs once as a factor in 24. It must, therefore, be used once in the l. c. m. The l. c. m., therefore, of 16 and 24 is $2 \times 2 \times 2 \times 2 \times 3$, or 48.

2. Find the l. c. m. of 6, 12, 16, and 24.

$$\begin{array}{r|l} 2 & 6 \quad 12 \quad 16 \quad 24 \\ 2 & 3 \quad 6 \quad 8 \quad 12 \\ 2 & 3 \quad 3 \quad 4 \quad 6 \\ & 3 \quad 1 \quad 2 \quad 3 \end{array}$$

Since 6 and 12 are exact divisors of 24, a multiple of 24 is also a multiple of 6 and 12, hence 6 and 12 may be rejected. Divide the remaining numbers by any prime factor that will divide *two or more of them.* In the same way divide the quotients until *no two of them* have a common divisor. 2 occurs 4 times in 16, hence it must be used 4 times in the l. c. m. 3 occurs once in 24, hence it must be used once in the l. c. m. Hence the l. c. m. of 6, 12, 16, and 24 is $2 \times 2 \times 2 \times 2 \times 3$ or 48.

Find the least common multiple of:

3. 15, 20, 30
4. 18, 24, 36
5. 14, 21, 42
6. 27, 54, 63
7. 32, 48, 96
8. 36, 54, 63
9. 64, 72, 108
10. 72, 84, 120
11. 54, 81, 135
12. 8, 12, 16, 20
13. 16, 24, 32
14. 20, 35, 42
15. 27, 45, 63
16. 28, 40, 56
17. 36, 48, 64
18. 32, 52, 65
19. 50, 60, 70
20. 55, 75, 88
21. 60, 72, 96
22. 84, 108, 120

Find the g. c. d. and the l. c. m. of each of the following:

23. 45, 105, 180
24. 36, 132, 192
25. 54, 108, 198
26. 39, 91, 130
27. 42, 70, 126
28. 48, 80, 144
29. 66, 134, 220
30. 60, 135, 180

FRACTIONS

1. Divide 1 into 5 equal parts; into 10 equal parts; into 15 equal parts.

2. What is meant by $\frac{1}{5}$, $\frac{1}{10}$, or $\frac{1}{15}$, of 1?

3. What is meant by $\frac{5}{10}$? by $\frac{3}{4}$ ft.? by $\frac{2}{7}$ yd.?

4. In 10 there are how many whole units?

An **integer** or an **integral number** is a whole number.

5. How many integral units are there in 5? in 10 ft.? in 5 yd.? in 4 mi.? in \$10?

6. If 1 ft. is divided into 12 equal parts, what is each part called? How many twelfths of a foot does it take to equal a foot?

A **fractional unit** is one of the equal parts into which an integral unit has been divided; as, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{12}$, etc.

A **fraction** is one or more fractional units; as, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{9}{10}$.

7. How many fractional units are there in $\frac{1}{12}$? in $\frac{3}{12}$? in $\frac{4}{12}$? in $\frac{5}{12}$? in $\frac{7}{12}$? in $\frac{9}{12}$? in $\frac{12}{12}$? How many are there in $\frac{1}{6}$? in $\frac{3}{6}$? in $\frac{5}{6}$? in $\frac{4}{6}$? in $\frac{6}{6}$?

8. Observe that the sum of the fractional units of an integral unit equals the integral unit. Thus, $\frac{12}{12}=1$; $\frac{5}{5}=1$; $\frac{4}{4}=1$; $\frac{6}{6}=1$; $\frac{10}{10}=1$.

9. In the fraction $\frac{3}{4}$, what is 3 called and what does it show?

10. In the fraction $\frac{3}{4}$, what is 4 called and what does it show?

Since the **denominator** (p. 23) of a fraction shows into how many equal parts the integral unit has been divided, as, tenths, thirds, fourths, it shows the *size* of the fractional unit.

Since the **numerator** (p. 23) of a fraction shows the number of fractional units taken, as, *two* thirds, $\frac{2}{3}$, it names the *number* of the fractional units.

11. Observe that the unit of measure in 12 yd. is 1 yd.; thus, 12 yd. means 12×1 yd; \$8 means $8 \times \$1$. What does 16 acres mean? 6 dozen?

12. Name the different units of measure in problem 11.

13. Observe that the unit of measure in $\frac{10}{12}$ in. is $\frac{1}{12}$ in.; thus, $\frac{7}{12}$ in. means $7 \times \frac{1}{12}$ in.; $\$\frac{4}{5}$ means $4 \times \$\frac{1}{5}$. What does $\frac{7}{8}$ acre mean? $\frac{5}{12}$ dozen?

14. What are the different units of measure in problem 13?

15. With what kind of a unit do we measure or compare whole numbers?

16. With what kind of a unit do we measure or compare fractions?

17. What is a proper fraction? (see p. 59). Explain why the fractional forms $\frac{3}{3}$, $\frac{8}{8}$, $\frac{10}{8}$, $\frac{5}{4}$, $\frac{4}{4}$, are not proper fractions.

18. What is an improper fraction? (see p. 59). A mixed number? (see p. 35).

REDUCTION OF FRACTIONS

Changing a mixed number to an improper fraction.

1. Change 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, each to the fractional unit 4ths; to 5ths; to 6ths; to 7ths. Thus, $1 = \frac{4}{4}$, $2 = \frac{8}{4}$, etc.

2. Change $1\frac{1}{2}$, $1\frac{3}{4}$, $2\frac{3}{5}$, $3\frac{5}{7}$, $2\frac{7}{8}$, $8\frac{3}{10}$, $9\frac{6}{11}$, $10\frac{5}{12}$, each to the fractional units indicated by the fraction of the mixed numbers. Thus, $1\frac{1}{2} = \frac{3}{2}$.

3. What kind of fractions are $\frac{3}{2}$, $\frac{7}{4}$, $\frac{13}{5}$?

Written Work

1. Change $11\frac{7}{15}$ to an improper fraction.

$11\frac{7}{15}$
15
165
7
172
15

Since 1 unit $= \frac{15}{15}$, 11 units $= 11 \times \frac{15}{15} = \frac{165}{15}$; $\frac{165}{15} + \frac{7}{15} = \frac{172}{15}$.

To change a mixed number to an improper fraction, ***multiply the whole number by the denominator. To this product add the numerator, and write the sum over the denominator.***

Change to improper fractions:

2. $15\frac{2}{3}$	12. $75\frac{8}{15}$	22. $83\frac{21}{62}$
3. $17\frac{3}{4}$	13. $40\frac{9}{17}$	23. $65\frac{17}{25}$
4. $18\frac{5}{6}$	14. $85\frac{5}{18}$	24. $28\frac{23}{39}$
5. $25\frac{5}{7}$	15. $92\frac{3}{19}$	25. $47\frac{20}{49}$
6. $35\frac{7}{8}$	16. $95\frac{7}{20}$	26. $75\frac{16}{71}$
7. $42\frac{7}{9}$	17. $48\frac{19}{30}$	27. $37\frac{59}{70}$
8. $51\frac{2}{11}$	18. $78\frac{3}{40}$	28. $46\frac{29}{55}$
9. $60\frac{5}{12}$	19. $80\frac{11}{50}$	29. $54\frac{27}{35}$
10. $45\frac{6}{13}$	20. $42\frac{14}{23}$	30. $82\frac{43}{51}$
11. $56\frac{9}{14}$	21. $71\frac{9}{34}$	31. $50\frac{21}{31}$

Changing an improper fraction to a mixed number.

1. Name the integers and the fractions in 5, $\frac{15}{2}$, $\frac{7}{4}$ ft., $\frac{64}{6}$ lb., \$20, $\frac{25}{5}$ yd., 10 oz., $\frac{28}{7}$ in.

2. How much greater than 1 are $\frac{4}{4}$? $\frac{5}{4}$? $\frac{8}{8}$? $\frac{11}{10}$? $\frac{15}{7}$? $\frac{12}{9}$? $\frac{16}{15}$? $\frac{10}{10}$?

3. In changing $\frac{7}{4}$ ft. to integers and fractions, think first how many fourths it takes to make one whole unit. Then $\frac{7}{4}$ = how many whole units and $\frac{3}{4}$ remaining?

Written Work

1. Change $\frac{145}{8}$ to a mixed number.

$8\overline{)145}$ $\quad 18\frac{1}{8}$

Since 1 = 8 eighths, in 145 eighths there are as many 1's as 8 eighths are contained times in 145 eighths, or $18\frac{1}{8}$. Therefore $\frac{145}{8} = 18\frac{1}{8}$.

Change to mixed numbers :

2. $\frac{25}{4}$	8. $\frac{86}{7}$	14. $\frac{57}{12}$
3. $\frac{37}{9}$	9. $\frac{65}{7}$ oz.	15. $\frac{75}{7}$ mi.
4. $\frac{46}{5}$	10. $\frac{75}{6}$	16. $\frac{98}{12}$ rd.
5. $\frac{29}{3}$	11. $\frac{99}{12}$ lb.	17. $\frac{69}{8}$ bu.
6. $\frac{65}{8}$	12. $\frac{148}{12}$ hr.	18. $\frac{110}{9}$ in.
7. $\frac{75}{9}$	13. $\frac{124}{11}$ min.	19. $\frac{116}{10}$ A.

Changing the size of fractional units without changing the value of the fraction.

1. $\frac{1}{2} = \frac{?}{8} = \frac{?}{16} = \frac{?}{24}$.

2. $\frac{2}{3} = \frac{?}{6} = \frac{?}{9} = \frac{?}{12} = \frac{?}{18}$.

3. $\frac{3}{4} = \frac{?}{8} = \frac{?}{12} = \frac{?}{16} = \frac{?}{20} = \frac{?}{24} = \frac{?}{28} = \frac{?}{32}$.

4. In changing $\frac{1}{2}$ to $\frac{3}{6}$, we multiply both terms of the fraction by 3. Does this affect the value of $\frac{1}{2}$? Draw a figure to show that $\frac{1}{2}$ and $\frac{3}{6}$ are the same in value.

5. In changing $\frac{3}{6}$ to $\frac{1}{2}$ we divide both terms of the fraction by 3. Does this change its value? Illustrate by the figure drawn for example 4.

Multiplying or dividing both terms of a fraction by the same number does not change its value.

6. Change $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{7}{12}$, $\frac{5}{8}$, $\frac{5}{12}$, each to twenty-fourths.

7. Change $\frac{4}{6}$, $\frac{10}{12}$, $\frac{15}{18}$, $\frac{16}{24}$, $\frac{10}{30}$, $\frac{18}{24}$, $\frac{25}{30}$, each to sixths.

8. Explain why changing the fractions in example 6 to twenty-fourths does not change the value of the fractions. Draw figures to illustrate.

9. Explain why changing the fractions in example 7 to sixths does not change the value of the fractions. Draw figures to illustrate.

Changing fractions to higher terms.

When several fractions are equal in value, the one having the largest numerator and denominator is said to have the *highest terms*, and the one having the smallest numerator and denominator is said to have the *lowest terms*.

1. In $\frac{1}{2} = \frac{4}{8} = \frac{8}{16} = \frac{12}{24}$, which fraction has the highest terms? the lowest terms?

2. Explain why the terms in $\frac{16}{24}$ are larger than in $\frac{2}{3}$?

3. In changing a fraction to *higher terms*, do we multiply or divide both terms of the fraction?

4. Explain the meaning of equivalent fractions.

5. Change $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{11}{12}$, to 48ths.

6. In changing $\frac{1}{2}$ to 8ths, into how many parts is the fractional unit $\frac{1}{2}$ divided? What is the *size* of each part? What are the *number* of parts?

Written Work

1. Change $\frac{3}{4}$ to 12ths.

$$\frac{3 \times 3 = 9}{4 \times 3 = 12}$$

Since $12 \div 4 = 3$, we must multiply both terms of the fraction by 3 to obtain a fraction of the same value with 12 for the denominator.

To change a fraction to higher terms *divide the required denominator by the denominator of the fraction, and multiply both terms by the quotient.*

2. Change to 40ths $\frac{4}{5}$, $\frac{3}{4}$, $\frac{5}{10}$, $\frac{5}{8}$, $\frac{3}{20}$.

3. Change to 56ths $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{14}$, $\frac{3}{7}$, $\frac{1}{2}$.

4. Change to 108ths $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{12}$, $\frac{5}{6}$, $\frac{4}{9}$.

5. $\frac{8}{9} = \frac{?}{36} = \frac{?}{45} = \frac{?}{108} = \frac{?}{99} = \frac{?}{81} = \frac{?}{72}$.

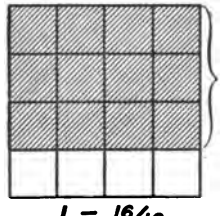
6. $\frac{5}{8} = \frac{?}{24} = \frac{?}{96} = \frac{?}{88} = \frac{?}{104}$.

7. $\frac{2}{3} = \frac{?}{63}$. 8. $\frac{3}{7} = \frac{?}{77}$. 9. $\frac{3}{4} = \frac{?}{144}$. 10. $\frac{5}{9} = \frac{?}{135}$.

Change:

11. $\frac{2}{3}$ to 36ths
12. $\frac{3}{4}$ to 48ths
13. $\frac{5}{6}$ to 60ths
14. $\frac{5}{12}$ to 96ths
15. $\frac{9}{11}$ to 121sts
16. $\frac{9}{16}$ to 128ths

Changing a fraction to its lowest terms.



1. In the square $\frac{12}{16} = \frac{?}{4}$; $\frac{8}{16} = \frac{?}{2}$; $\frac{10}{16} = \frac{?}{8}$.

2. $\frac{4}{16} = \frac{?}{4}$; $\frac{8}{16} = \frac{?}{8}$.

3. Draw figures to show that $\frac{9}{12} = \frac{3}{4}$; $\frac{14}{16} = \frac{7}{8}$; $\frac{6}{9} = \frac{2}{3}$.

4. Which do you prefer, $\frac{15}{20}$ of a dollar or $\frac{3}{4}$ of a dollar? Explain why.

5. By what number do we divide both terms of $\frac{15}{20}$ to change the fraction to $\frac{3}{4}$?

6. When $\frac{8}{16}$ is changed to $\frac{4}{8}$, is it in its lowest terms? Express it in its lowest terms.

7. Change to lowest terms $\frac{2}{4}$, $\frac{6}{8}$, $\frac{9}{12}$, $\frac{8}{12}$, $\frac{10}{15}$, $\frac{15}{18}$, $\frac{7}{12}$, $\frac{9}{10}$, $\frac{18}{24}$, $\frac{14}{21}$. Observe that in changing these fractions to the lowest terms, both terms of the fraction are divided by their *greatest common divisor*.

8. Explain by drawing figures why dividing both terms of a fraction by the same number does not change its value.

Written Work

1. Change $\frac{36}{42}$ to its lowest terms.

$36 \div 2 = 18 \div 3 = 6$
$42 \div 2 = 21 \div 3 = 7$

Or

g. c. d. $= 6$
$36 \div 6 = 6$
$42 \div 6 = 7$

Since the value of a fraction is not changed by dividing both terms by the same number, we divide both numerator and denominator first by 2; then we divide the terms of the resulting fraction $\frac{18}{21}$ by 3. Since 6 and 7 have no common divisor except 1, $\frac{6}{7}$ is in the lowest terms. Or we may, in one step, divide both terms of the fraction by their g. c. d., 6.

To change a fraction to its lowest terms *cancel all common factors from both terms or divide both terms by their greatest common divisor.*

Change to lowest terms:

2. $\frac{12}{24}$	5. $\frac{28}{60}$	8. $\frac{84}{108}$	11. $\frac{48}{100}$	14. $\frac{160}{180}$
3. $\frac{24}{36}$	6. $\frac{18}{40}$	9. $\frac{120}{144}$	12. $\frac{120}{360}$	15. $\frac{144}{210}$
4. $\frac{26}{42}$	7. $\frac{60}{96}$	10. $\frac{182}{192}$	13. $\frac{56}{144}$	16. $\frac{175}{875}$

Changing to similar fractions.

1. What are similar fractions?

2. To what *common denominator* may you change $\frac{3}{4}$ and $\frac{4}{5}$ to make them similar? $\frac{3}{4} = \frac{?}{20}$; $\frac{4}{5} = \frac{?}{20}$.

Change to similar fractions:

3. $\frac{1}{4}, \frac{1}{8}$	7. $\frac{9}{11}, \frac{2}{3}$	11. $\frac{5}{12}, \frac{7}{8}$	15. $\frac{5}{6}, \frac{4}{9}$
4. $\frac{3}{4}, \frac{9}{12}$	8. $\frac{5}{9}, \frac{7}{8}$	12. $\frac{5}{8}, \frac{7}{9}$	16. $\frac{11}{12}, \frac{4}{5}$
5. $\frac{7}{8}, \frac{5}{6}$	9. $\frac{3}{4}, \frac{7}{8}$	13. $\frac{5}{6}, \frac{3}{8}$	17. $\frac{4}{11}, \frac{6}{7}$
6. $\frac{7}{10}, \frac{4}{5}$	10. $\frac{5}{6}, \frac{4}{9}$	14. $\frac{5}{9}, \frac{5}{12}$	18. $\frac{4}{15}, \frac{9}{60}$

NOTE. — $\frac{1}{4}$ and $\frac{1}{3}$ may be changed to 12ths or 24ths, and in either case have common denominators, but when changed to 12ths, they are expressed in their *least common denominator* (l. c. d.).

Changing fractions to their least common denominator.

1. What is the least common multiple of 2, 3, and 4?

2. What is the l. c. m. of the denominators of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$?

3. Change the fractions $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$ to equivalent fractions having the l. c. m. of the denominators for a like denominator. How, then, do you change fractions to their least common denominator?

Written Work

1. Change $\frac{3}{4}$, $\frac{7}{10}$, and $\frac{2}{5}$ to similar fractions having the least common denominator.

$2)\overline{4\ \ 10\ \ 5}$

$2\ \ 5$

l. c. d. $= 2 \times 2 \times 5 = 20$

$\frac{3 \times 5}{4 \times 5} = \frac{15}{20}$

$\frac{2 \times 4}{5 \times 4} = \frac{8}{20}$

$\frac{7 \times 2}{10 \times 2} = \frac{14}{20}$

In finding the least common multiple we may reject 5, since 10 is a multiple of 5.

The least common multiple of the denominators 4 and 10 is $2 \times 2 \times 5$, or 20, which is the least common denominator of the given fractions.

To change $\frac{3}{4}$ to 20ths divide the required denominator by the given denominator and multiply both terms by the quotient. Proceed in the same way with the other fractions.

Change to similiar fractions having the l. c. d.:

2. $\frac{1}{2}$, $\frac{1}{3}$
3. $\frac{1}{4}$, $\frac{1}{6}$
4. $\frac{1}{5}$, $\frac{1}{4}$
5. $\frac{2}{3}$, $\frac{3}{4}$
6. $\frac{5}{6}$, $\frac{3}{8}$
7. $\frac{9}{10}$, $\frac{3}{5}$
8. $\frac{5}{8}$, $\frac{7}{12}$
9. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$
10. $\frac{1}{6}$, $\frac{2}{3}$, $\frac{3}{4}$
11. $\frac{1}{2}$, $\frac{3}{8}$, $\frac{4}{5}$
12. $\frac{3}{4}$, $\frac{2}{5}$, $\frac{3}{7}$
13. $\frac{7}{8}$, $\frac{2}{9}$, $\frac{3}{10}$
14. $\frac{5}{12}$, $\frac{4}{5}$, $\frac{2}{3}$
15. $\frac{4}{7}$, $\frac{1}{2}$, $\frac{7}{9}$
16. $\frac{9}{16}$, $\frac{7}{20}$
17. $\frac{5}{9}$, $\frac{5}{12}$
18. $\frac{5}{6}$, $\frac{4}{9}$, $\frac{10}{12}$
19. $\frac{2}{11}$, $\frac{3}{5}$, $\frac{1}{2}$
20. $\frac{12}{18}$, $\frac{7}{24}$, $\frac{9}{32}$
21. $\frac{20}{28}$, $\frac{3}{4}$, $\frac{4}{7}$
22. $\frac{5}{13}$, $\frac{4}{7}$, $\frac{9}{26}$
23. $\frac{3}{15}$, $\frac{5}{8}$, $\frac{3}{4}$
24. $\frac{1}{12}$, $\frac{3}{4}$, $\frac{2}{3}$
25. $\frac{1}{20}$, $\frac{1}{2}$, $\frac{3}{5}$
26. $\frac{6}{7}$, $\frac{3}{4}$, $\frac{1}{3}$
27. $\frac{8}{9}$, $\frac{1}{7}$, $\frac{1}{2}$
28. $\frac{5}{6}$, $\frac{4}{7}$, $\frac{1}{5}$

ADDITION OF FRACTIONS

1. Observe the different kinds of units in each of the following numbers:

5, 10 ft., 4 bu., 7, 9 ft., 8 oz., $6, 12 oz., 10, 6 bu.

2. Add the numbers that have units of the same kind.

3. Why can you not add 10 feet and 4 bushels?

4. What change must you make in 3 bushels, 3 pints, and 3 quarts before you can add them?

5. Can you add $\frac{3}{4}$ and $\frac{5}{4}$ without change? Can you add them after changing both to 20ths?

6. What change must be made in fractions whose denominators are not alike before they can be added?

Written Work

Adding fractions.

1. Add $\frac{3}{4}$ and $\frac{5}{9}$.

$36 = \text{l. c. d.}$

$\frac{3}{4} = \frac{27}{36}$

$\frac{5}{9} = \frac{20}{36}$

$\frac{3}{4} + \frac{5}{9} = \frac{47}{36}$ or $1\frac{11}{36}$

The l. c. d. is 36; $\frac{3}{4} = \frac{27}{36}$ and $\frac{5}{9} = \frac{20}{36}$; $\frac{27}{36} + \frac{20}{36} = \frac{47}{36}$, or $1\frac{11}{36}$.

Observe the three steps in adding fractions:

1. If necessary, make the fractions similar, that is, change them to a common denominator.

2. Write the sum of the numerators over the common denominator.

3. Change the sum to its simplest form.

Add:

2. $\frac{3}{4}$, $\frac{7}{9}$

3. $\frac{7}{9}$, $\frac{3}{8}$

4. $\frac{6}{7}$, $\frac{4}{5}$

5. $\frac{4}{9}$, $\frac{5}{8}$

6. $\frac{5}{10}$, $\frac{2}{3}$

7. $\frac{5}{6}$, $\frac{7}{9}$

8. $\frac{2}{11}$, $\frac{4}{5}$

9. $\frac{9}{10}$, $\frac{7}{8}$

10. $\frac{2}{3}$, $\frac{3}{4}$, $\frac{1}{2}$

11. $\frac{2}{3}$, $\frac{6}{7}$, $\frac{5}{6}$

12. $\frac{6}{7}$, $\frac{1}{9}$

13. $\frac{3}{4}$, $\frac{4}{5}$, $\frac{1}{2}$

14. $\frac{2}{3}$, $\frac{7}{8}$, $\frac{3}{4}$

15. $\frac{4}{5}$, $\frac{6}{7}$

16. $\frac{2}{7}$, $\frac{1}{3}$, $\frac{3}{4}$

17. $\frac{4}{9}$, $\frac{1}{2}$, $\frac{2}{5}$

18. $\frac{7}{8}$, $\frac{7}{18}$, $\frac{8}{9}$

19. $\frac{11}{12}$, $\frac{4}{5}$, $\frac{9}{10}$

Adding mixed numbers.

1. Add $12\frac{3}{4}$, $6\frac{3}{5}$, and $10\frac{7}{8}$.

$40 = \text{l. c. d.}$

$12\frac{3}{4} = 12\frac{30}{40}$

$6\frac{3}{5} = 6\frac{24}{40}$

$10\frac{7}{8} = 10\frac{35}{40}$

$12\frac{3}{4} + 6\frac{3}{5} + 10\frac{7}{8} = 28\frac{89}{40} = 30\frac{9}{40}$.

The l. c. m. of 4, 5, and 8 is 40, which is the least common denominator of the given fractions. $\frac{3}{4} = \frac{30}{40}$; $\frac{3}{5} = \frac{24}{40}$; $\frac{7}{8} = \frac{35}{40}$. The sum of the fractions is $\frac{89}{40}$, which added to the sum of the whole numbers $= 30\frac{9}{40}$.

Add:

2. $1\frac{5}{6} + \frac{3}{5} + 2\frac{7}{8}$
3. $10\frac{8}{9} + 12\frac{3}{8} + 5\frac{3}{27}$
4. $2\frac{5}{12} + 5\frac{3}{4} + 9\frac{4}{15}$
5. $1\frac{3}{4} + 7\frac{4}{5} + 8\frac{7}{8}$
6. $9\frac{3}{5} + 16\frac{7}{10} + 5\frac{14}{15}$
7. $4\frac{7}{8} + \frac{11}{16} + \frac{13}{18}$
8. $3\frac{1}{2} + 4\frac{3}{4} + 1\frac{7}{12}$
9. $4\frac{2}{3} + 5\frac{7}{10} + 9\frac{5}{18}$
10. $12\frac{2}{3} + \frac{3}{5} + \frac{7}{8} + 3\frac{9}{10}$
11. $2\frac{2}{9} + 1\frac{1}{6} + 3\frac{1}{4}$

12. Mr. Seward traveled $19\frac{1}{2}$ miles on Monday, $21\frac{3}{4}$ miles on Tuesday, and $22\frac{5}{8}$ miles on Wednesday. How far did he travel in the three days?

13. John deposited $\$6\frac{1}{5}$ in a savings bank, James $\$7\frac{7}{10}$, Henry $\$9\frac{3}{4}$, and Joseph $\$11\frac{7}{25}$. How much did they all deposit?

14. A dry-goods merchant sold a customer $16\frac{3}{4}$ yards of silk, $24\frac{3}{8}$ yards of flannel, and $22\frac{1}{2}$ yards of muslin. How many yards of goods did he sell?

15. A merchant bought a barrel of flour for $\$3\frac{3}{5}$, a barrel of pork for $\$11\frac{7}{10}$, and a barrel of sugar for $\$15\frac{3}{4}$. What was the amount of his bill?

16. Four boys weigh, respectively, $90\frac{1}{2}$ pounds, $95\frac{1}{4}$ pounds, $98\frac{7}{8}$ pounds, and $101\frac{9}{16}$ pounds. What is their entire weight?

17. The distance from A to B is $13\frac{4}{5}$ miles; from B to C $20\frac{7}{8}$ miles; from C to D $31\frac{3}{10}$ miles. What is the distance from A to D?

SUBTRACTION OF FRACTIONS

1. How do we add like fractions?

2. What change must be made in unlike fractions before they can be added?

Since in *adding* like fractions we find the *sum* of the numerators, in *subtracting* like fractions we take the *difference* of the numerators; thus, $\frac{5}{8} + \frac{3}{8} = \frac{8}{8}$, sum; $\frac{5}{8} - \frac{3}{8} = \frac{2}{8}$, difference.

Written Work

Find the value of:

1. $7\frac{1}{4} - 2\frac{5}{8}$.

$8 =$ l. c. d.

$7\frac{1}{4} = 6\frac{5}{4} = 6\frac{10}{8}$

$2\frac{5}{8} = \quad 2\frac{5}{8}$

$7\frac{1}{4} - 2\frac{5}{8} = 4\frac{5}{8}$.

Since $\frac{5}{8}$ is greater than $\frac{1}{4}$, change $7\frac{1}{4}$ to $6\frac{5}{4}$. The l. c. d. is 8. $\frac{5}{4} = \frac{10}{8}$, $\frac{5}{8} = \frac{5}{8}$. Subtracting $2\frac{5}{8}$ from $6\frac{10}{8}$, we find that the remainder is $4\frac{5}{8}$.

Name the three steps in subtracting fractions.

2. $\frac{6}{7} - \frac{4}{5}$
3. $3\frac{1}{3} - 1\frac{1}{2}$
4. $5 - 1\frac{2}{3}$
5. $5\frac{1}{4} - 2\frac{2}{5}$
6. $10\frac{3}{8} - 5\frac{3}{4}$
7. $20\frac{1}{12} - 13\frac{5}{6}$
8. $7\frac{5}{9} - 2\frac{1}{3}$
9. $50\frac{1}{5} - 30\frac{1}{3}$
10. $6\frac{3}{5} - 2\frac{3}{7}$
11. $31\frac{9}{16} - 20\frac{5}{28}$
12. $11\frac{3}{5} - 5\frac{2}{3}$
13. $15 - 9\frac{9}{13}$
14. $65\frac{4}{15} - 63\frac{9}{20}$
15. $44\frac{3}{4} - 12\frac{1}{2}$
16. $36\frac{14}{35} - 11\frac{4}{15}$
17. $27\frac{4}{5} - 19\frac{4}{11}$
18. $81\frac{5}{6} - 14\frac{6}{7}$
19. $\frac{3}{5} - \frac{3}{7}$

REVIEW OF FRACTIONS

Find the value of:

1. $\frac{2}{3} + \frac{3}{4} - \frac{5}{6}$
2. $3\frac{1}{2} + 2\frac{1}{4} - 1\frac{5}{8}$
3. $5\frac{1}{3} + 4\frac{5}{6} - 4\frac{5}{12}$
4. $1\frac{5}{8} + 2\frac{1}{12} - 1\frac{5}{6}$
5. $9\frac{9}{15} - 4\frac{2}{5} + 3\frac{7}{10}$
6. $12\frac{3}{4} - 5\frac{1}{2} + 2\frac{2}{9}$
7. $\frac{3}{4} - \frac{1}{2} + \frac{7}{8} + \frac{17}{18}$
8. $20 + \frac{1}{16} + \frac{1}{12} - 2\frac{1}{4}$

9. $\frac{1}{3}+\frac{1}{4}+\frac{5}{6}-\frac{7}{12}$

10. $18\frac{11}{24}-10\frac{3}{8}+5\frac{1}{3}+2\frac{5}{6}$

11. $15-3\frac{2}{3}+4\frac{1}{5}$

12. $40+60+30\frac{11}{18}$

13. $19\frac{37}{32}-11\frac{5}{8}+1\frac{5}{16}$

14. $7\frac{7}{9}+4\frac{1}{3}+11\frac{5}{18}$

15. $3\frac{5}{12}+9\frac{5}{9}-7\frac{7}{24}$

16. $22\frac{2}{11}+7\frac{1}{8}-20\frac{10}{33}$

17. $8\frac{3}{4}+22\frac{2}{5}+43\frac{3}{8}$

18. $225\frac{3}{8}+132\frac{5}{12}+80\frac{17}{20}$

19. $12\frac{2}{3}+19\frac{7}{8}-27\frac{5}{6}$

20. $19\frac{1}{5}+11\frac{3}{8}+14\frac{1}{3}$

21. A merchant bought a bale of linen for \$68, and paid $\$1\frac{1}{4}$ for transportation. The linen being damaged, he sold it for $\$49\frac{3}{4}$. How much did he lose?

22. Arthur bought a hat and gave in payment a ten-dollar bill. If he received in change $\$7\frac{3}{4}$, how much did the hat cost?

23. A contractor agreed to build a house for \$1500, but failed to get his pay by \$100. He paid out $\frac{1}{5}$ of the contract price for work, and $\frac{2}{5}$ for material. How much did he make?

24. From $160\frac{3}{4}$ acres of land, lots of $18\frac{1}{2}$ acres, $20\frac{3}{4}$ acres, $31\frac{3}{8}$ acres, $42\frac{1}{2}$ acres, and $25\frac{5}{16}$ acres were sold. How many acres remained unsold?

25. The treasurer of a literary society received $\$492\frac{1}{2}$. He spent for light and heat $\$50\frac{3}{4}$, for new books $\$77\frac{1}{4}$, for a lecturer's expenses $\$26\frac{1}{2}$, and for music for an entertainment \$80. How much remained in his hands?

26. From the sum of $8\frac{1}{2}$ and $7\frac{3}{4}$ take $9\frac{5}{6}$.

27. Two boys earn $\$15\frac{3}{4}$ in a week. If one earns $\$7\frac{1}{2}$, how much does the other earn?

28. From a bin of sugar containing $427\frac{3}{4}$ pounds, $28\frac{1}{2}$ pounds were sold at one time, 31 pounds at another, and 50 pounds at another. How many pounds remained to be sold?

29. A cab driver drove $30\frac{1}{4}$ miles on a certain day. From his start to his stable it was $1\frac{7}{8}$ miles and he made that trip twice. What was the number of miles for which he was paid?

30. A tub of butter weighed $80\frac{7}{8}$ pounds; the box alone weighed $4\frac{5}{8}$ pounds. How much did the butter weigh?

31. Five floors of an office building are each 12 feet high, two floors are each $13\frac{3}{4}$ feet high, and one floor is $18\frac{5}{6}$ feet high. What is the total height of the floors?

32. In one piece of serge there are $25\frac{3}{8}$ yards, in a second piece $18\frac{3}{4}$ yards, in a third piece $22\frac{9}{16}$ yards, in a fourth piece $27\frac{1}{2}$ yards. How many yards are there in the four pieces?

33. The widths of 4 lots are as follows: $30\frac{1}{2}$ feet, $42\frac{5}{10}$ feet, $38\frac{11}{20}$ feet, and $48\frac{7}{12}$ feet. Find the entire width of the lots?

34. From a web of muslin containing $45\frac{9}{16}$ yards, were sold $28\frac{7}{8}$ yards. How many yards remained unsold?

35. A grain dealer buys two loads of corn, one containing $64\frac{3}{4}$ bushels, the other $50\frac{1}{2}$ bushels. To one man he sells 20 bushels of this corn, to another $13\frac{1}{4}$ bushels, to another $7\frac{1}{2}$ bushels. How many bushels has he left after these sales?

36. A traveler went $16\frac{1}{2}$ miles the first day, $17\frac{3}{4}$ miles the second day, and $18\frac{7}{8}$ miles the third day. How far did he travel in the three days?

37. A ball team spent $\$2\frac{1}{2}$ for balls, \$40 for uniforms, $\$3\frac{3}{4}$ for bats, and $\$18\frac{1}{4}$ for gloves and mitts. How much did the team's outfit cost?

38. Change to lowest terms $\frac{3}{8}\frac{2}{4}\frac{8}{2}$.

39. A teacher's salary was \$95 per month. He spent \$20 for board, \$7$\frac{1}{2}$ for room, \$9$\frac{3}{5}$ for clothes, and \$2$\frac{1}{10}$ for other expenses. How much did he save out of his month's salary?

40. A dealer bought 61$\frac{3}{4}$ bushels of apples from one man; 127$\frac{1}{2}$ bushels from another; 89$\frac{3}{4}$ bushels from another. How much did they cost at \$.75 per bushel?

41. A student spends $\frac{1}{4}$ of the day in study, $\frac{1}{6}$ in recitations, $\frac{1}{12}$ at his meals, and $\frac{1}{8}$ in recreation and exercise. What part of the day has he left for sleep?

42. A boy walked around a field 60$\frac{1}{2}$ rods long and 40 rods wide. How far did he walk?

43. The income from an apartment house for a month is as follows: 2 suites of rooms each \$28$\frac{1}{2}$, 2 suites each \$32, and 2 suites each \$34.50. What is the total income and what is the average per suite?

44. William had \$45$\frac{1}{2}$; he gave his sister \$37$\frac{1}{5}$. How much had he remaining?

45. Mr. Mellon owned $\frac{1}{8}$ of an oil well. Later he bought $\frac{7}{16}$ of it. What part of the well did he not own?

46. A clerk earns \$40 a month and spends for board \$13$\frac{1}{2}$, for clothes \$7$\frac{3}{4}$, and for other expenses \$14$\frac{4}{5}$. How much of his month's salary does he save?

47. From a piece of ribbon containing 12 yards were sold $\frac{1}{2}$ yard, 1$\frac{1}{4}$ yards, $\frac{3}{4}$ yard, $\frac{5}{8}$ yard, 2$\frac{3}{4}$ yards, and 3 yards. How many yards remained unsold?

48. A mail carrier works 8 hours per day. In making 3 deliveries he works 1$\frac{1}{6}$ hours, 2$\frac{1}{3}$ hours, and 2$\frac{3}{4}$ hours. How much time has he for a fourth delivery?

49. $\frac{2}{5}$ of a pole is in the ground, $\frac{2}{9}$ of it in the water, and the rest in the air. What part of the pole is in the air?

50. From a piece of cloth containing $100\frac{1}{2}$ yards, were sold $20\frac{3}{4}$ yards, $28\frac{5}{8}$ yards, and $31\frac{1}{4}$ yards, respectively. How many yards remained?

51. A fisherman caught 3 fish. The first weighed $\frac{1}{2}$ pound, the second $\frac{7}{8}$ pound, and the third $1\frac{1}{4}$ pounds. How much did the 3 fish weigh?

52. A real estate agent bought $16\frac{1}{2}$ acres of land; he sold $2\frac{1}{4}$ acres to one man, and $5\frac{7}{8}$ acres to another. How many acres had he remaining?

53. The sum of two fractions is $\frac{7}{8}$, and one of the fractions is $\frac{3}{5}$. What is the other?

54. Robert had $\$6\frac{4}{5}$ and his father gave him $\$5\frac{3}{4}$. How much had he then?

55. The McJunkin dairy sold milk as follows: Monday $65\frac{3}{8}$ gallons, Tuesday $60\frac{1}{2}$ gallons, Wednesday $71\frac{3}{4}$ gallons, Thursday 69 gallons, Friday $67\frac{7}{8}$ gallons, and Saturday $90\frac{1}{4}$ gallons. What were the total sales for the week?

56. A salesman traveled in 4 days as follows: Monday $122\frac{1}{2}$ miles, Tuesday $187\frac{3}{4}$ miles, Wednesday $93\frac{1}{8}$ miles, and Thursday $207\frac{3}{16}$ miles. How many miles did he travel in all?

57. A grocery bill is as follows: flour $\$2\frac{4}{5}$, oranges $\$\frac{3}{5}$, lettuce $\$\frac{1}{4}$, celery $\$\frac{7}{20}$, sugar $\$1$, dried peaches $\$\frac{1}{4}$, canned tomatoes $\$\frac{1}{2}$, and preserves $\$\frac{3}{5}$. What is the amount of the bill?

58. If a boat will carry safely 800 pounds, how many pounds of provisions can be carried when three men whose weights are, respectively, $165\frac{1}{2}$ pounds, $182\frac{3}{4}$ pounds, and 208 pounds, are in the boat?

59. If a merchant's collections in 4 days are $\$128\frac{3}{5}$, $\$192\frac{3}{4}$, $\$280\frac{1}{20}$, and $\$164\frac{4}{5}$, what is his average collection per day?

MULTIPLICATION OF FRACTIONS

Multiplying a fraction by an integer.

1. 4×4 days means 4 days are taken 4 times as an addend; thus, 4 days + 4 days + 4 days + 4 days = 16 days. By multiplication 4×4 days = 16 days. Explain in the same way what $4 \times \$\frac{1}{4}$ means.

Multiplication is a *short* form of addition.

2. In the problem $8 \times \frac{1}{3}$ days $= \frac{8}{3}$ days $= 2\frac{2}{3}$ days, what term of the fraction is multiplied by 8? What is $8 \times \frac{2}{3}$ days? $8 \times \frac{4}{5}$ rods? $10 \times \frac{3}{4}$ pounds?

3. In $8 \times \frac{1}{4}$ hour do we *increase* or *decrease* the *number* of fractional units? Do we change the *size* of the fractional units? Observe, then, that a fraction can be multiplied by an integer by *multiplying the numerator* of the fraction by the integer.

4. $5 \times \frac{1}{10}$ hour $= \frac{1}{2}$ hour. What term of the fraction divided by 5 changes the fraction to $\frac{1}{2}$?

5. Compare $\frac{1}{10}$ and $\frac{1}{2}$ in *number* of fractional units. In *size* of the fractional units. Is $\frac{1}{2}$, 5 times $\frac{1}{10}$ in size?

6. In $5 \times \frac{1}{10} = \frac{1}{2}$, what change is made in the *number* of fractional units? in the *size* of the fractional units? Observe, then, that a fraction can also be multiplied by an integer by *dividing the denominator* of the fraction by the integer.

Give products:

7. $8 \times \frac{3}{24}$	**11.** $5 \times \frac{4}{5}$	**15.** $12 \times \frac{5}{36}$
8. $6 \times \frac{5}{18}$	**12.** $7 \times \frac{8}{9}$	**16.** $9 \times \frac{4}{27}$
9. $15 \times \frac{7}{45}$	**13.** $8 \times \frac{4}{7}$	**17.** $9 \times 1\frac{1}{12}$
10. $11 \times \frac{2}{9}$	**14.** $10 \times 1\frac{1}{12}$	**18.** $10 \times \frac{4}{40}$

Written Work

1. Multiply $\frac{1}{12}$ by 3.

$$3 \times \frac{1}{12} = \frac{\cancel{3} \times 1}{\cancel{12}} = \frac{1}{4}.$$

Since multiplying the numerator of a fraction by an integer multiplies the fraction by the integer, $3 \times \frac{1}{12} = \frac{3}{12}$, or $\frac{1}{4}$.

A fraction is multiplied by an integer by multiplying the numerator or by dividing the denominator of the fraction by the integer.

NOTE.—Shorten the work, when possible, by cancellation.

Multiply:

2.	$16 \times \frac{3}{8}$	7.	$12 \times \frac{11}{3}$	12.	$25 \times \frac{8}{15}$	17.	$16 \times \frac{15}{24}$
3.	$24 \times \frac{7}{8}$	8.	$13 \times \frac{12}{26}$	13.	$15 \times \frac{14}{21}$	18.	$27 \times \frac{44}{81}$
4.	$27 \times \frac{2}{3}$	9.	$14 \times \frac{11}{12}$	14.	$10 \times \frac{12}{25}$	19.	$33 \times \frac{72}{132}$
5.	$45 \times \frac{4}{9}$	10.	$12 \times \frac{17}{18}$	15.	$21 \times \frac{24}{63}$	20.	$45 \times \frac{55}{135}$
6.	$18 \times \frac{20}{6}$	11.	$11 \times \frac{7}{22}$	16.	$36 \times \frac{55}{144}$	21.	$36 \times \frac{91}{144}$

22. At $\$\frac{3}{5}$ per dozen, find the cost of 12 dozen oranges.

23. Find the cost of 230 histories at $\$\frac{1}{2}$ each.

24. Find the cost of 250 readers at $\$\frac{3}{16}$ each.

25. Find the cost of 36 yards of carpet at $\$\frac{3}{4}$ per yard.

26. Find the cost of digging a ditch 125 rods long at $\$\frac{7}{20}$ per rod.

27. Multiply $2\frac{3}{4}$ by 12.

$$\begin{array}{r} 2\frac{3}{4} \\ \hline 12 \\ \hline 9 = 12 \times \frac{3}{4} \\ \hline 24 \\ \hline 33 \end{array}$$

First we multiply the fraction and then the integer by the multiplier. Then we add the results.

Find the value of:

28. $9 \times 2\frac{1}{4}$	**33.** $7 \times 8\frac{1}{2}$	**38.** $27 \times 12\frac{5}{36}$
29. $12 \times 3\frac{2}{3}$	**34.** $10 \times 2\frac{7}{15}$	**39.** $19 \times 8\frac{1}{3}$
30. $18 \times 5\frac{3}{8}$	**35.** $16 \times 9\frac{5}{12}$	**40.** $26 \times 5\frac{7}{39}$
31. $22 \times 4\frac{3}{10}$	**36.** $24 \times 8\frac{2}{7}$	**41.** $36 \times 7\frac{1}{8}$
32. $21 \times 2\frac{5}{14}$	**37.** $32 \times 7\frac{13}{40}$	**42.** $42 \times 8\frac{1}{12}$

43. At $\$3\frac{3}{10}$ a day, how much will a man earn in 26 days?

44. Find the cost of 18 tons of hay at $\$15\frac{3}{4}$ per ton.

45. Find the cost of 48 suits for boys at $\$4\frac{2}{5}$ each.

46. Find the cost of 18 dozen spoons at $\$7\frac{1}{2}$ a dozen.

47. Find the cost of 10 pairs of shoes at $\$2\frac{3}{4}$ a pair.

48. A gardener sold 275 crates of strawberries averaging $\$2\frac{4}{5}$, 168 crates of blackberries averaging $\$1\frac{3}{4}$, 200 crates of raspberries averaging $\$2\frac{1}{4}$. How much did he receive for them?

49. A laborer earns $\$2\frac{1}{5}$ per day and his son earns $\$1\frac{3}{10}$ per day. Find their total earnings for 310 days.

50. A New York State apple grower sold 320 barrels of apples at $\$1\frac{1}{5}$ per barrel; 200 barrels at $\$2\frac{1}{4}$ per barrel; and 372 barrels at $\$1\frac{3}{5}$ per barrel. Find the amount received for them.

51. Find the cost of labor and material for fencing a farm 100 rods long and 60 rods wide, at $\$1\frac{3}{5}$ per rod.

52. A street-car conductor averages 10 hours per day for 30 days. Find his monthly wages at $22\frac{1}{2}¢$ per hour.

53. An automobile party travels $135\frac{3}{4}$ miles Monday, $176\frac{5}{16}$ miles Tuesday, $193\frac{9}{16}$ miles Wednesday, $96\frac{3}{8}$ miles Thursday, and $185\frac{11}{24}$ miles Friday. If the cost of the trip is $11\frac{5}{16}¢$ per mile, find the total cost.

54. If a department store buys 300 boys' suits at $\$5\frac{3}{4}$ and retails them at \$7.98, what is the gain?

55. One newsboy averages $23\frac{5}{16}¢$ per day for 320 days, another averages $36\frac{4}{15}¢$ per day for 300 days, and a third $39\frac{4}{11}¢$ per day for 297 days. Find their total earnings.

56. James walks $1\frac{7}{8}$ miles to school each morning. How many miles does he walk to and from school if he attends 160 days in the term?

57. How much will 8 pounds of pork cost @ $11\frac{3}{4}¢$?

58. A father bought 5 pairs of skates for his children. If they cost $\$2\frac{1}{4}$ per pair, how much did the five pairs cost?

59. A track-laying gang lay $4\frac{3}{8}$ miles of track in a week. How many miles can they lay in 12 weeks?

60. A grocer bought 3 tubs of butter weighing, respectively, 27 pounds, 31 pounds, and 24 pounds at $11\frac{1}{2}$ cents a pound. How much did all cost?

61. His average price realized on the sale of this butter was $17\frac{1}{2}$ cents per pound. How much did he make on the transaction?

62. When lead is worth $\$5\frac{1}{5}$ per hundred pounds, how much will 1800 pounds cost?

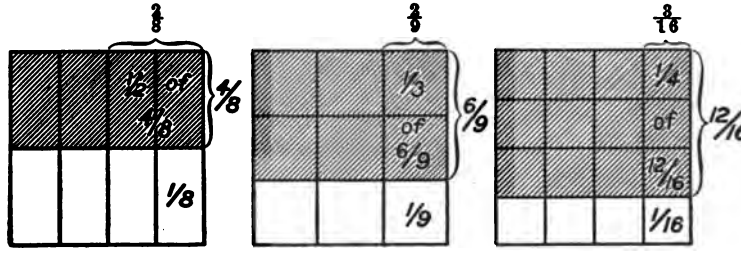
63. If creamery butter is worth 27 cents a pound and country butter $24\frac{1}{2}$ cents a pound, how much will 20 pounds of the former and 25 pounds of the latter cost?

64. In one day the receipts of coffee at New York were 30,250 bags. Each bag contained 135 pounds. How much was it worth at $6\frac{1}{5}$ cents per pound?

65. When pig iron is worth $\$16\frac{7}{10}$ per ton, how much will 42 tons cost?

66. When raisins are worth $6\frac{3}{4}$ cents a pound, how much will 50 pounds cost?

Finding a fractional part of a fraction, or multiplying a fraction by a fraction.



$\frac{1}{2}$ of $\frac{4}{8} = \frac{2}{8}$ $\frac{1}{3}$ of $\frac{6}{9} = \frac{2}{9}$ $\frac{1}{4}$ of $\frac{12}{16} = \frac{3}{16}$

1. In finding $\frac{1}{2}$ of $\frac{4}{8}$, into how many equal parts are $\frac{4}{8}$ divided? How many parts are taken?

2. Show that $\frac{4}{8}$ can be separated into 2 equal parts without changing the *size* of the fractional unit.

3. In finding $\frac{1}{3}$ of $\frac{6}{9}$, into how many equal parts is $\frac{6}{9}$ divided? Is the *size* of the fractional unit changed?

4. In finding $\frac{1}{4}$ of $\frac{12}{16}$, into how many equal parts is $\frac{12}{16}$ divided? Is the *size* of the fractional unit increased?

5. Since $\frac{1}{3}$ of $\frac{6}{9} = \frac{2}{9}$, what is $\frac{2}{3}$ of $\frac{6}{9}$?

Since $\frac{1}{3}$ of $\frac{6}{9} = \frac{2}{9}$, $\frac{2}{3}$ of $\frac{6}{9} = 2 \times \frac{2}{9}$ or $\frac{4}{9}$.

Find:

6. $\frac{1}{3}$ of $\frac{3}{4}$

7. $\frac{2}{3}$ of $\frac{3}{4}$

8. $\frac{1}{5}$ of $\frac{10}{15}$

9. $\frac{2}{5}$ of $\frac{10}{15}$

10. $\frac{4}{5}$ of $\frac{10}{15}$

11. $\frac{1}{8}$ of $\frac{16}{20}$

12. $\frac{3}{8}$ of $\frac{16}{20}$

13. $\frac{7}{8}$ of $\frac{16}{20}$

14. $\frac{1}{6}$ of $\frac{12}{18}$

15. $\frac{5}{6}$ of $\frac{12}{18}$

16. $\frac{3}{4}$ of $\frac{12}{18}$

17. $\frac{7}{12}$ of $\frac{12}{20}$

18. $\frac{8}{12}$ of $\frac{8}{12}$

19. $\frac{9}{10}$ of $\frac{20}{25}$

20. $\frac{10}{10}$ of $\frac{20}{25}$

21. Separate $\frac{12}{15}$ into 3 equal parts.

22. To what equivalent fraction can $\frac{4}{5}$ be changed so as to separate it into 3 equal parts; that is, take $\frac{1}{3}$ of $\frac{4}{5}$?

23. Is $\frac{12}{15}$ the same in value as $\frac{4}{5}$?

24. Separate $\frac{12}{15}$ into three equal parts; then does $\frac{1}{3}$ of $\frac{4}{5}$ $= \frac{4}{15}$?

25. Since *one* third of $\frac{4}{5} = \frac{4}{15}$, what do *two* thirds of $\frac{4}{5}$ equal? $\frac{2}{3}$ of $\frac{4}{5} = ?$ $\frac{2}{5}$ of $\frac{3}{5} = ?$

$$\frac{1}{3} \times \frac{4}{5} = \frac{1 \times 4}{3 \times 5} = \frac{4}{15}; \quad \frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}; \quad \frac{2}{5} \times \frac{3}{5} = \frac{2 \times 3}{5 \times 5} = \frac{6}{25}.$$

Observe that in each case the result is obtained by multiplying the numerators together for a new numerator and the denominators for a new denominator.

$\frac{2}{3}$ of $\frac{4}{5}$ means that we are to divide $\frac{4}{5}$ into 3 equal parts and take two of these parts. The operation is the same as in $\frac{2}{3} \times \frac{4}{5}$, which is called **multiplying a fraction by a fraction.**

A fractional part of a fraction equals the fraction times the fraction.

26. A man owns $\frac{2}{3}$ of a mill, and sells $\frac{1}{3}$ of his share. What part of the mill does he sell?

27. A man travels $\frac{4}{5}$ of his journey the first day, and the second day $\frac{3}{4}$ as far as the first day. What part of the journey does he travel the second day?

28. Mr. Todd owns $\frac{3}{4}$ of a store, and sells $\frac{4}{5}$ of his share to Mr. Craig. What part of the store does Mr. Craig buy?

29. How much will $\frac{3}{4}$ of a gallon of molasses cost at \$ $\frac{4}{5}$ a gallon?

30. Alice had $\frac{3}{4}$ of a pound of candy, and gave away $\frac{4}{5}$ of it. What part of a pound did she give away?

31. Edwin had $\frac{3}{25}$ of a dollar, and spent $\frac{1}{4}$ of it. What part of a dollar had he remaining?

Written Work

1. Find $\frac{4}{5}$ of $\frac{7}{8}$, or multiply $\frac{4}{5}$ by $\frac{7}{8}$.

$$\frac{4}{5} \text{ of } \frac{7}{8} = \frac{4 \times 7}{5 \times 8} = \frac{28}{40} = \frac{7}{10}$$

Since *one* fifth of $\frac{7}{8} = \frac{7}{5 \times 8}$,
four fifths of $\frac{7}{8} = \frac{4 \times 7}{5 \times 8} = \frac{28}{40} = \frac{7}{10}$

Or canceling,

$$\frac{4}{5} \text{ of } \frac{7}{8} = \frac{\cancel{4} \times 7}{5 \times \cancel{8}_{2}} = \frac{7}{10}$$

The* product of two fractions *is found by multiplying the numerators for a new numerator and the denominators for a new denominator.

Change mixed numbers to improper fractions.

The word "of" between two fractions is equivalent to the sign (×) of multiplication.

$\frac{2}{3} \times 6$ is called multiplication, but in reality 6 cannot be taken $\frac{2}{3}$ times as an addend. The fact is, 6 is partitioned into 3 parts and 2 of these parts are taken.

In the problem $\frac{3}{4}$ of 7, 7 may be expressed as an improper fraction $\frac{7}{1}$. Therefore $\frac{3}{4}$ of $7 = \frac{3}{4}$ of $\frac{7}{1} = \frac{21}{4} = 5\frac{1}{4}$.

Find:

2. $\frac{2}{3}$ of $\frac{5}{6}$	**8.** $\frac{25}{36}$ of $\frac{8}{15}$	**14.** $\frac{15}{32}$ of $\frac{44}{35}$
3. $\frac{3}{5}$ of $\frac{5}{7}$	**9.** $\frac{9}{10}$ of $\frac{20}{12}$	**15.** $\frac{26}{45}$ of $\frac{55}{19}$
4. $\frac{3}{4}$ of $\frac{5}{6}$	**10.** $\frac{11}{8}$ of $\frac{10}{17}$	**16.** $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{3}{4}$
5. $\frac{2}{3}$ of $\frac{7}{9}$	**11.** $\frac{15}{16}$ of $\frac{8}{4}$	**17.** $\frac{9}{10}$ of $\frac{25}{39}$ of $\frac{13}{15}$
6. $\frac{5}{6}$ of $\frac{3}{8}$	**12.** $\frac{16}{19}$ of $\frac{17}{24}$	**18.** $\frac{7}{8}$ of $\frac{30}{32}$ of $\frac{24}{35}$
7. $\frac{3}{5}$ of $\frac{3}{9}$	**13.** $\frac{20}{7}$ of $\frac{49}{8}$	**19.** $\frac{25}{49}$ of $\frac{105}{32}$ of $\frac{36}{65}$

20. Mr. Weldin, who owned $\frac{3}{4}$ of a store, sold $\frac{2}{7}$ of his share. What part of the store did he sell?

21. The first of two lines measures $\frac{13}{16}$ inches, and the second is $\frac{7}{8}$ as long. How long is the second line?

Multiplying a mixed number by a mixed number.

Written Work

1. Multiply $3\frac{3}{8}$ by $8\frac{4}{5}$.

$3\frac{3}{8} = \frac{27}{8}$; $8\frac{4}{5} = \frac{44}{5}$; $\frac{27}{8} \times \frac{44}{5} = \frac{297}{10} = 29\frac{7}{10}$.

Multiply:

2. $1\frac{1}{2} \times 1\frac{1}{3}$	5. $1\frac{3}{8} \times 2\frac{5}{9}$	8. $20\frac{1}{4} \times 5\frac{7}{9}$
3. $2\frac{1}{2} \times 3\frac{2}{3}$	6. $6\frac{1}{3} \times 2\frac{1}{4}$	9. $12\frac{4}{5} \times 1\frac{7}{8}$
4. $2\frac{3}{4} \times 3\frac{5}{6}$	7. $16\frac{2}{3} \times 5\frac{1}{7}$	10. $39\frac{3}{5} \times 66\frac{2}{3}$

11. There are $16\frac{1}{2}$ feet in a rod. How many feet are there in $30\frac{7}{8}$ rods?

12. Find the cost of $92\frac{3}{4}$ pounds of paper @ $3\frac{2}{5}¢$.

13. When turkeys are selling at $11\frac{3}{4}$ cents per pound, how much will be realized from a flock weighing $165\frac{3}{4}$ pounds?

14. A shoe dealer bought $10\frac{1}{2}$ dozen rubbers at $\$4\frac{3}{4}$ per dozen. How much did they cost him?

15. A city lot containing $721\frac{1}{4}$ square feet was sold at the rate of $\$5\frac{4}{5}$ per square foot. For how much was the lot sold?

16. If hay is worth $\$16\frac{1}{4}$ per ton, how much will 3 loads of hay weighing respectively $1\frac{3}{20}$ tons, $1\frac{3}{10}$ tons, and $\frac{9}{10}$ tons cost?

17. If a soldier is allowed $\frac{3}{8}$ lb. of meat per day, how much meat will it take to supply 280 men for $6\frac{1}{2}$ days?

18. When cheese is worth $18\frac{3}{4}$ cents per pound at wholesale, how much will a $30\frac{1}{4}$-pound cheese cost?

19. The freight on iron ore from Conneaut, O., to Pittsburg, Pa., is $\$.65$ per ton. What is the freight on a car load of ore weighing $48\frac{11}{20}$ tons?

20. In a certain building, during January, $181\frac{9}{10}$ tons of coal were consumed. If the coal cost $\$2\frac{1}{4}$ per ton, what was the total expense for coal?

21. How much is $11\frac{1}{12}$ ounces of gold worth @ $\$18\frac{3}{4}$?

22. A farmer sold $37\frac{1}{4}$ bushels of apples at $\$\frac{3}{4}$ per bushel. How much did he get for them?

23. If strawberries were sold for $\$2\frac{1}{4}$ per crate, how much was realized from the sale of $27\frac{1}{2}$ crates?

24. If the strawberries in example 23 were sold for $\$3\frac{1}{5}$ per crate, what was the gain?

25. A coal miner in a month digs $112\frac{3}{5}$ tons of coal. At $\$\frac{12}{25}$ per ton, how much does he earn?

26. When hay is selling at $\$\frac{1}{5}$ per hundred pounds, find the cost of 5 bales, their weights being $1\frac{1}{8}$ cwt., $1\frac{1}{4}$ cwt., $1\frac{1}{5}$ cwt., $1\frac{1}{10}$ cwt., and $1\frac{3}{20}$ hundredweight.

27. A coal dealer sold 1850 tons of coal. $\frac{3}{5}$ of his sales were anthracite coal, the rest bituminous. The former he sold for $\$6\frac{1}{4}$ per ton, the latter for $\$2\frac{1}{4}$ per ton. How much were his total sales worth?

28. If sound travels $373\frac{1}{3}$ yards in 1 second, how far away is a locomotive if you see the steam from its whistle 5 seconds before you hear its sound?

29. A school is open $5\frac{3}{4}$ hours each day. How many hours is it open in a month of 20 school days? in a term of $8\frac{1}{2}$ months?

30. How much will $9\frac{3}{4}$ bushels of huckleberries cost at $\$3\frac{1}{4}$ per bushel?

31. When venison steak is worth $\$.40$ per pound, how much will $6\frac{1}{4}$ pounds cost?

DIVISION OF FRACTIONS

Dividing a fraction by an integer.

1. If 10 marbles are divided equally between two boys, how many marbles does each boy receive? Then $10 \div 2$ means that 10 is to be separated into two equal parts, 5 and 5, and that one of these parts is to be taken.

2. If $\frac{6}{8}$ of a yard of ribbon is divided equally between 2 girls, how many eighths of a yard does each girl receive? Then $\frac{6}{8} \div 2$ means that $\frac{6}{8}$ is divided into 2 equal parts, $\frac{3}{8}$ and $\frac{3}{8}$, and that one of these parts is to be taken.

3. If $\$\frac{24}{25}$ is divided equally among four boys, what part of the money does each boy receive? Then $\frac{24}{25} \div 4$ means that $\frac{24}{25}$ is to be separated into 4 equal parts, $\frac{6}{25}$, $\frac{6}{25}$, $\frac{6}{25}$, and $\frac{6}{25}$, and that one of these parts is to be taken.

4. What *term* of the fraction $\frac{6}{8}$ do you divide by 2 to get $\frac{3}{8}$?

5. What *term* of the fraction $\frac{24}{25}$ do you divide by 4 to get $\frac{6}{25}$?

6. Show that the fractions $\frac{4}{8}$, $\frac{6}{10}$, $\frac{12}{15}$, and $\frac{8}{10}$ each can be divided by 2 without changing the *size* of the fractional unit. Observe, then, that a fraction can be divided by an integer, *by dividing the numerator* of the fraction by the integer.

Divide:

7. $\frac{4}{5}$ by 2
8. $\frac{8}{10}$ by 4
9. $\frac{12}{15}$ by 3
10. $\frac{15}{20}$ by 5
11. $\frac{18}{20}$ by 6
12. $\frac{24}{25}$ by 8
13. $\frac{21}{25}$ by 7
14. $\frac{32}{35}$ by 8
15. $\frac{45}{49}$ by 9
16. $\frac{108}{125}$ by 12
17. $\frac{99}{100}$ by 11
18. $\frac{96}{100}$ by 12
19. $\frac{54}{60}$ by 9
20. $\frac{60}{75}$ by 10
21. $\frac{121}{180}$ by 11

22. In $\frac{4}{5} \div 5$, are there 5 equal parts of $\frac{1}{5}$ each? Can we change $\frac{4}{5}$ to the equivalent fraction $\frac{20}{25}$? $\frac{20}{25} \div 5 = \frac{4}{25}$. Since $\frac{4}{5} = \frac{20}{25}$, then $\frac{4}{5} \div 5 = \frac{4}{25}$.

23. In dividing $\frac{4}{5}$ by 5, what *term* of the fraction $\frac{4}{5}$ did we multiply by 5? Observe, then, that a fraction can be divided by an integer *by multiplying the denominator* of the fraction by the integer.

Divide:

24. $\frac{3}{4}$ by 4	**27.** $\frac{7}{8}$ by 3	**30.** $\frac{4}{7}$ by 3
25. $\frac{7}{8}$ by 2	**28.** $\frac{4}{5}$ by 5	**31.** $\frac{5}{8}$ by 6
26. $\frac{5}{6}$ by 3	**29.** $\frac{4}{9}$ by 3	**32.** $\frac{5}{9}$ by 4

33. If 5 men do $\frac{10}{12}$ of a piece of work in one day, what part of the work is done by one man?

34. John earns $\$\frac{4}{5}$ in 4 hours. How much does he receive per hour?

35. A real estate agent buys $\frac{3}{4}$ of an acre of land and divides it into 10 lots. What is the size of each lot?

36. When 4 bushels of wheat sell for $3\frac{1}{5}$ dollars, what is the price per bushel?

SUGGESTION. — First change $3\frac{1}{5}$ to an improper fraction.

37. Four newsboys earn $\$3\frac{2}{5}$ by selling papers, and share equally. How much does each boy receive?

38. A conductor earned $\$3\frac{1}{5}$ in 16 hours. How much was that per hour?

39. Thirteen carpenters were paid $\$45\frac{1}{2}$ for doing a piece of work. How much did each receive?

40. It cost $\$1\frac{1}{5}$ to send a cablegram of 6 words. How much was that per word?

Written Work

1. Divide $\frac{24}{35}$ by 6.

$$\frac{24}{35} \div 6 = \frac{24 \div 6}{35} = \frac{4}{35}$$

Since dividing the numerator of a fraction by a whole number divides the fraction by that number, we divide the numerator of $\frac{24}{35}$ by 6. The result is $\frac{4}{35}$.

2. Divide $\frac{2}{3}$ by 7.

$$\frac{2}{3} \div 7 = \frac{2}{3 \times 7} = \frac{2}{21}$$

Since multiplying the denominator of a fraction by a whole number divides the fraction by that number, we multiply the denominator of $\frac{2}{3}$ by 7. The result is $\frac{2}{21}$.

3. Divide $\frac{12}{13}$ by 18.

$$\frac{12}{13} \div 18 = \frac{\overset{2}{\cancel{12}}}{13 \times \underset{3}{\cancel{18}}} = \frac{2}{39}$$

In cases like this it is convenient to indicate the multiplication of the denominator by the integer, and cancel.

The factor 6 is contained twice in the numerator and three times in the denominator. The result is $\frac{2}{39}$.

NOTE. — When the numerator is exactly divisible by the integer, the method of example 1 is preferable; in all other cases the methods of examples 2 and 3.

To divide a fraction by an integer, *divide the numerator or multiply the denominator by the integer, canceling when possible.*

Find quotients:

4. $\frac{16}{17} \div 8$	9. $\frac{88}{89} \div 19$	14. $\frac{68}{11} \div 12$
5. $\frac{6}{7} \div 5$	10. $\frac{20}{27} \div 14$	15. $\frac{56}{13} \div 14$
6. $\frac{18}{23} \div 6$	11. $\frac{18}{41} \div 11$	16. $\frac{18}{30} \div 27$
7. $\frac{28}{33} \div 7$	12. $\frac{17}{40} \div 34$	17. $\frac{12}{19} \div 36$
8. $\frac{15}{16} \div 10$	13. $\frac{21}{26} \div 14$	18. $\frac{19}{25} \div 38$

Dividing a small mixed number by an integer.

Written Work

1. Divide $16\frac{7}{8}$ by 7.

$16\frac{7}{8} \div 7 = \frac{135}{8} \div 7 = \frac{135}{56} = 2\frac{23}{56}$

First change the mixed number to an improper fraction. Then divide by multiplying the denominator. The result is $\frac{135}{56}$, which may be reduced to the mixed number $2\frac{23}{56}$.

Find the quotients:

2. $22\frac{3}{4} \div 13$	5. $41\frac{7}{8} \div 22$	8. $51\frac{3}{7} \div 20$
3. $35\frac{1}{6} \div 16$	6. $37\frac{1}{3} \div 10$	9. $29\frac{2}{11} \div 9$
4. $17\frac{3}{5} \div 12$	7. $18\frac{1}{4} \div 32$	10. $46\frac{2}{5} \div 15$

11. A train travels $99\frac{3}{10}$ miles in 4 hours. What is its rate per hour?

12. When a man earns $\$24\frac{2}{5}$ in 6 days, how much does he earn per day?

13. A ten-story building is $112\frac{3}{5}$ feet high. What is the average height of each floor?

14. In 28 days a hotel used $361\frac{3}{4}$ pounds of butter. How many pounds did it use a day?

Dividing a large mixed number by an integer.

Written Work

1. Divide $1570\frac{3}{4}$ by 8.

$8\overline{)1570\frac{3}{4}}$

$196\frac{11}{32}$

Divide as in whole numbers. The quotient is 196 and the remainder $2\frac{3}{4}$. $\frac{1}{8}$ of $2\frac{3}{4} = \frac{1}{8}$ of $\frac{11}{4}$, or $\frac{11}{32}$. The result is $196\frac{11}{32}$.

Find the value of:

2. $2687\frac{7}{9} \div 9$	5. $4267\frac{3}{4} \div 12$	8. $1977\frac{5}{7} \div 5$
3. $1365\frac{8}{11} \div 8$	6. $5831\frac{2}{3} \div 11$	9. $3164\frac{8}{11} \div 10$
4. $3853\frac{5}{8} \div 7$	7. $4783\frac{3}{5} \div 6$	10. $1790\frac{7}{10} \div 12$

Dividing any number by a fraction by changing both numbers to similar fractions.

1. What kind of whole units can be added? What kind can be subtracted? Illustrate by examples.

2. What change must be made in 10 ft. ÷ 2 in. before the numbers can be divided?

3. Observe that only concrete numbers of the same *kind* can be divided. Thus \$20 ÷ \$10, 100 ft. ÷ 10 ft., etc.

4. What name is given to fractions having the same denominator? Change $\frac{4}{5}$ and $\frac{3}{4}$ to 20ths.

5. How often is 15 twentieths contained in 16 twentieths? 16 twentieths ÷ 15 twentieths = $1\frac{1}{15}$. **Test:** 15 twentieths × $1\frac{1}{15}$ = 16 twentieths. Then $\frac{4}{5} \div \frac{3}{4} = \frac{16}{20} \div \frac{15}{20} = 16 \div 15 = 1\frac{1}{15}$.

6. $8 \div \frac{4}{5} = \frac{8}{1} \div \frac{4}{5} = \frac{40}{5} \div \frac{4}{5} = ?$

NOTE. — Observe that 8 can be written as an improper fraction $\frac{8}{1}$ and then both fractions can be changed to similar fractions.

7. How, then, can a fraction be divided by a fraction?

NOTE. — Mixed numbers should first be changed to improper fractions.

Written Work

Find quotients:

1. $\frac{7}{8} \div \frac{3}{8}$
2. $\frac{10}{15} \div \frac{7}{15}$
3. $\frac{5}{6} \div \frac{7}{8}$
4. $\frac{7}{12} \div \frac{5}{12}$
5. $\frac{3}{4} \div \frac{7}{8}$
6. $\frac{5}{9} \div \frac{3}{10}$
7. $\frac{3}{7} \div \frac{5}{14}$
8. $\frac{12}{15} \div \frac{3}{4}$
9. $1\frac{2}{3} \div \frac{3}{4}$
10. $5\frac{4}{5} \div 2\frac{5}{6}$
11. $3\frac{3}{4} \div 4$
12. $5\frac{7}{8} \div 10$
13. Compare $1 \div \frac{1}{4}$ and $1 \div \frac{2}{4}$.
14. Compare $8 \div \frac{1}{3}$ and $8 \div \frac{2}{3}$.
15. How many times are $2\frac{3}{4}$ contained in $8\frac{1}{4}$? Divide $8\frac{1}{4}$ by $2\frac{3}{4}$.

Dividing any number by a fraction by inverting the divisor and multiplying.

1. $1 \div \frac{1}{4}$ = how many?
2. $1 \div \frac{2}{4}$ = how many?
3. $1 \div \frac{1}{6}$ = how many?
4. $1 \div \frac{2}{6}$ = how many?
5. $1 \div \frac{3}{6}$ = how many?
6. $1 \div \frac{1}{8}$ = how many?

Observe that each fraction is contained in 1 as many times as the *numerator* of the fraction is *contained in the denominator.*

7. $3 \div \frac{1}{4}$ = how many?
8. $4 \div \frac{1}{4}$ = how many?
9. $3 \div \frac{1}{6}$ = how many?
10. $4 \div \frac{3}{6}$ = how many?

Since $\frac{1}{4}$ is contained in 1, $\frac{4}{1}$ or 4 times, $\frac{1}{4}$ is contained in 3, $3 \times \frac{4}{1} = \frac{12}{1}$, or 12 times. A short method of dividing any number by a fraction is to multiply the given number by the *number of times the fraction is contained in* 1.

Since $1 \div \frac{1}{3} = 3$, then $1 \div \frac{2}{3}$ must equal $\frac{1}{2}$ of 3 or $\frac{3}{2}$. Divide the denominator of $\frac{2}{3}$ by the numerator. Show that $1 \div \frac{2}{3} = 1 \times \frac{3}{2}$. Observe, then, that the number of times any fraction is contained in 1 equals the fraction *inverted;* thus, $\frac{3}{4}$ is contained in 1, $\frac{4}{3}$ times.

11. Divide 8 by $\frac{2}{4}$.

$\frac{2}{4}$ is contained in 1, $\frac{4}{2}$ times. In 8 it is contained $8 \times \frac{4}{2}$ times. $8 \times \frac{4}{2} = \frac{8 \times 4}{2} = 16$.

12. State the two methods of dividing fractions. Show that improper fractions and whole and mixed numbers are divided on the same principles as proper fractions.

Find quotients:

13. $4 \div \frac{1}{3}$
14. $1\frac{1}{5} \div \frac{2}{3}$
15. $5 \div \frac{3}{4}$
16. $1\frac{1}{6} \div \frac{5}{6}$
17. $8 \div \frac{7}{8}$
18. $7 \div \frac{4}{5}$
19. $10 \div \frac{2}{5}$
20. $1\frac{1}{4} \div \frac{1}{4}$
21. $12 \div \frac{6}{7}$

Written Work

1. Divide $\frac{2}{3}$ by $\frac{4}{5}$.

$$\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{\cancel{4}_{2}} = \frac{5}{6}.$$

Since $1 \div \frac{4}{5} = \frac{5}{4}$, $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3}$ of $\frac{5}{4}$. Canceling, the result is $\frac{5}{6}$.

2. Divide $5\frac{4}{5}$ by 8.

$$5\frac{4}{5} \div 8 = \frac{29}{5} \times \frac{1}{8} = \frac{29}{40}$$

1 divided by 8 equals $\frac{1}{8}$; $5\frac{4}{5}$, or $\frac{29}{5}$, divided by 8, equals $\frac{29}{5}$ times $\frac{1}{8}$, or $\frac{29}{40}$.

3. Divide $3\frac{5}{6}$ by $4\frac{2}{3}$.

$$3\frac{5}{6} \div 4\frac{2}{3} = \frac{23}{\cancel{6}_{2}} \times \frac{\cancel{3}}{14} = \frac{23}{28}$$

1 divided by $4\frac{2}{3}$, or $\frac{14}{3}$, equals $\frac{3}{14}$; $3\frac{5}{6}$, or $\frac{23}{6}$, divided by $4\frac{2}{3}$, equals $\frac{23}{6}$ times $\frac{3}{14}$ or $\frac{69}{84}$.

Find the quotients:

4. $13 \div \frac{5}{7}$	10. $\frac{11}{12} \div \frac{5}{9}$	16. $\frac{81}{112} \div \frac{21}{32}$
5. $25 \div \frac{7}{8}$	11. $\frac{12}{13} \div \frac{6}{11}$	17. $\frac{49}{108} \div \frac{63}{64}$
6. $121 \div \frac{11}{12}$	12. $\frac{7}{15} \div \frac{1}{5}$	18. $\frac{25}{36} \div \frac{15}{32}$
7. $256 \div \frac{16}{21}$	13. $\frac{9}{16} \div \frac{1}{4}$	19. $\frac{5}{22} \div \frac{8}{8}$
8. $42 \div \frac{5}{9}$	14. $\frac{9}{16} \div \frac{1}{9}$	20. $\frac{14}{25} \div \frac{7}{5}$
9. $55 \div \frac{6}{11}$	15. $\frac{24}{65} \div \frac{8}{13}$	21. $\frac{27}{56} \div \frac{33}{35}$

Change both numbers to fractions; then invert the divisor and multiply.

NOTES. — 1. When possible, cancellation should be used.

2. In solving problems that contain fractions, sometimes the method of changing to like fractional units is the shorter.

22. $2\frac{1}{2} \div 2\frac{1}{3}$	28. $18\frac{2}{7} \div 7\frac{1}{9}$	34. $67\frac{3}{16} \div 7\frac{3}{4}$
23. $12\frac{1}{2} \div 16\frac{2}{3}$	29. $19\frac{3}{8} \div 1\frac{3}{28}$	35. $88\frac{2}{21} \div 16\frac{2}{3}$
24. $8\frac{2}{3} \div 4\frac{3}{4}$	30. $160 \div \frac{5}{8}$	36. $30\frac{3}{16} \div 5\frac{3}{4}$
25. $5\frac{1}{5} \div 4\frac{1}{4}$	31. $16\frac{3}{7} \div 6\frac{9}{11}$	37. $95\frac{5}{6} \div 7\frac{2}{3}$
26. $7\frac{3}{8} \div 6\frac{1}{2}$	32. $77 \div 2\frac{1}{8}$	38. $178\frac{1}{3} \div \frac{5}{18}$
27. $8\frac{4}{5} \div 4\frac{1}{4}$	33. $103 \div 10\frac{3}{10}$	39. $200 \div 6\frac{1}{4}$

40. When common laborers earn $1.75 in a day of $9\frac{1}{2}$ hours, how much do they earn per hour?

41. The product of two numbers is $21\frac{7}{8}$. One of the numbers is $4\frac{3}{7}$, what is the other?

42. The material for a dress, containing $11\frac{1}{2}$ yards of cloth, cost $\$18\frac{3}{5}$. What was the cost per yard?

43. I invested $\$44\frac{4}{5}$ in books at $\$3\frac{1}{5}$ per volume. How many volumes did I buy?

44. Duffy ran 100 yards in $9\frac{3}{5}$ seconds. What was his rate per second?

45. Dan Patch paced a mile in $116\frac{1}{4}$ seconds. How many yards did he cover in a second?

46. At $\$\frac{3}{2\frac{3}{5}}$ apiece, how many tablets can be bought for $\$\frac{3}{5}$.

47. By the Pennsylvania Railroad the distance from Pittsburg to Cleveland is 150 miles. A "fast" train runs the distance in $3\frac{1}{2}$ hours. What is the rate of the train per hour?

48. When cement is worth $\$3\frac{1}{5}$ per barrel, how many barrels can be bought for $\$102\frac{2}{5}$?

49. At $\$1\frac{1}{4}$ each, how many baseballs can be purchased for $\$22\frac{1}{2}$?

50. There are $412\frac{1}{2}$ grains of silver in a silver dollar. How many dollars can be made from 9900 grains?

51. A franc is worth $19\frac{3}{10}$ cents. How many francs are equal to $38\frac{2}{5}$ cents?

52. How many barrels, each holding $2\frac{3}{4}$ bushels, can be filled from $899\frac{1}{4}$ bushels of apples?

53. Into how many pieces, each $1\frac{1}{2}$ yards long, can a piece of ribbon 12 yards long be cut?

54. How many cans, each holding $2\frac{1}{2}$ gallons, can be filled from a barrel of oil containing 50 gallons?

SHORT METHODS

Learn the following parts of $1:

$6\frac{1}{4}$ cents $= \frac{1}{16}$ of $1.	$37\frac{1}{2}$ cents $= \frac{3}{8}$ of $1.
$8\frac{1}{3}$ cents $= \frac{1}{12}$ of $1.	50 cents $= \frac{1}{2}$ of $1.
$12\frac{1}{2}$ cents $= \frac{1}{8}$ of $1.	$62\frac{1}{2}$ cents $= \frac{5}{8}$ of $1.
$16\frac{2}{3}$ cents $= \frac{1}{6}$ of $1.	$66\frac{2}{3}$ cents $= \frac{2}{3}$ of $1.
20 cents $= \frac{1}{5}$ of $1.	75 cents $= \frac{3}{4}$ of $1.
25 cents $= \frac{1}{4}$ of $1.	$83\frac{1}{3}$ cents $= \frac{5}{6}$ of $1.
$33\frac{1}{3}$ cents $= \frac{1}{3}$ of $1.	$87\frac{1}{2}$ cents $= \frac{7}{8}$ of $1.

1. Find the cost of 32 pounds of rice @ $6\frac{1}{4}¢$ a pound.

$6\frac{1}{4}¢ = \$\frac{1}{16}$. Since at $1 a pound, 32 lb. would cost $32, at $\frac{1}{16}$ a pound, they will cost $\frac{1}{16}$ of $32, or $2.

Find, by a short method, the cost of the following articles:

2. 24 ft. hose @ $8\frac{1}{3}¢$.

3. 40 rd. fence @ $12\frac{1}{2}¢$.

4. 36 yd. challis @ $16\frac{2}{3}¢$.

5. 35 gal. molasses @ $20¢$.

6. 44 qt. strawberries @ $25¢$.

7. 21 yd. flannel @ $33\frac{1}{3}¢$.

8. 48 neckties @ $37\frac{1}{2}¢$.

9. 30 shirts @ $50¢$.

10. 72 gal. sirup @ $12\frac{1}{2}¢$.

11. 150 yd. oilcloth @ $66\frac{2}{3}¢$.

12. 36 doz. collars @ $75¢$.

13. 64 bu. wheat @ $87\frac{1}{2}¢$.

14. At $6\frac{1}{4}¢$ per gal., how many gallons of oil can be bought for $3?

15. $\$3 \div \$\frac{1}{16} = 3 \times \frac{16}{1} = 48$, the number of gallons.

Since $6\frac{1}{4}¢ = \$\frac{1}{16}$, as many gallons can be bought as $\$\frac{1}{16}$ is contained times in $3, which is 48 times. Hence 48 gallons can be bought.

16. At $12\frac{1}{2}¢$ per yd., how many yards of ribbon can be bought for $25¢$? for $50¢$? for $1.50? for $3? for $5?

17. At $8\frac{1}{3}¢$ per lb., how many pounds of raisins can be bought for $1? for $5? for $2.50?

18. At 25 cents each, how many fishing lines can be bought for $.50? for $1? $1.50?

19. At $37\frac{1}{2}¢$ a gallon, how many gallons of molasses can be bought for $2? for $4?

20. At $33\frac{1}{3}¢$ per pair, how many pairs of stockings can be bought for $1? for $3? for $5? for $15?

21. At 25¢ per meal, how many persons can be fed for $2? for $5? for $9? for $20?

22. At $16\frac{2}{3}¢$ per gallon, how many gallons of gasoline can be bought for $.50? for $2? for $6? for $9?

23. 25 cents is contained how often in $1? how often in $5?

24. At 25 cents a pound, how many pounds of butter can be bought for $5?

25. At $62\frac{1}{2}$ cents a yard, how many yards of serge can be bought for $1.25?

26. At $33\frac{1}{3}$ cents a pound, how many pounds of butter can be bought for $4? for $6? for $10?

Divide the following at sight:

27. $8 by 25 cents.
28. $11 by $33\frac{1}{3}$ cents.
29. $18 by 50 cents.
30. $27 by $16\frac{2}{3}$ cents.
31. $12 by $8\frac{1}{3}$ cents.
32. $32 by 20 cents.
33. $40 by $6\frac{1}{4}$ cents.
34. $50 by $12\frac{1}{2}$ cents.
35. $72 by 25 cents.
36. $90 by $33\frac{1}{3}$ cents.

FRACTIONAL PARTS OF NUMBERS

Finding what part one number is of another.

1. What part of 1 is 2? What part of 4 is 3? of 6 is 5? Express the answers also in the form of division. Thus, $\frac{1}{2} = 1 \div 2$; $\frac{3}{4} = 3 \div 4$; $\frac{5}{6} = 5 \div 6$.

2. What part of $\frac{3}{5}$ is $\frac{2}{5}$? of $\frac{5}{6}$ is $\frac{4}{6}$? $\frac{2}{5} \div \frac{3}{5} = 2 \div 3 = \frac{2}{3}$; $\frac{4}{6} \div \frac{5}{6} = 4 \div 5 = \frac{4}{5}$. How, then, can you find what part one number is of another?

Divide the smaller number by the larger number.

Written Work

1. What part of 108 is 48?

$$48 \div 108 = \frac{48}{108} = \frac{4}{9}.$$

We divide the smaller number 48 by 108, which gives the fraction $\frac{48}{108}$. This reduced to its lowest terms equals $\frac{4}{9}$.

2. What part of $4\frac{1}{2}$ is $\frac{3}{4}$?

$$4\frac{1}{2} = \frac{9}{2}, \quad \frac{3}{4} \div \frac{9}{2} = \frac{\cancel{3}}{\cancel{4}_{2}} \times \frac{2}{\cancel{9}_{3}} = \frac{1}{6}$$

We reduce the mixed number to the improper fraction $\frac{9}{2}$. We then divide $\frac{3}{4}$ by $\frac{9}{2}$ as in example 1, page 173. The result is $\frac{1}{6}$.

What part of:

3. 48 is 36?
4. 96 is 16?
5. 117 is 104?
6. 144 is 128?
7. $\frac{4}{5}$ is $\frac{3}{5}$?
8. $1\frac{1}{2}$ is $1\frac{1}{4}$?

9. If John had 60 cents and spent 20 cents, what part of his money did he spend?

10. The distance between two stations is 72 miles. What part of the distance are 18 miles?

11. What part of a yard are 18 inches?

12. Ruth draws a line on the blackboard $\frac{1}{2}$ a foot long and Martha draws one $\frac{7}{8}$ of a foot long. What part of the length of Martha's line is the length of Ruth's?

Finding a number when the value of a fractional part of the number is given.

1. What is the number of which 4 is $\frac{1}{2}$?

If $4 = \frac{1}{2}$ of the number, $\frac{2}{2}$ of the number, or the number, $= 2 \times 4$, or 8.

2. 6 is $\frac{2}{3}$ of what number?

If 6 is *two* thirds of a number, *one* third of the number equals *one half* of 6, or 3, and *three* thirds of it or the number equals 3×3, or 9.

3. 8 is $\frac{4}{5}$ of what number? **5.** 21 is $\frac{7}{8}$ of what number?

4. 9 is $\frac{3}{4}$ of what number? **6.** 64 is $\frac{8}{9}$ of what number?

Written Work

1. 240 is $\frac{3}{4}$ of what number?

$240 \div \frac{3}{4} = \overset{80}{\cancel{240}} \times \frac{4}{\cancel{3}} = 320.$

Since 240 is $\frac{3}{4}$ of a number, $\frac{1}{4}$ of the number equals $\frac{1}{3}$ of 240, and $\frac{4}{4}$ of it equals $\frac{4}{3}$ of 240, which is 320.

Divide the number by the fractional part.

2. 965 is $\frac{5}{8}$ of what number? **5.** 909 is $\frac{9}{10}$ of what number?

3. 872 is $\frac{8}{9}$ of what number? **6.** 672 is $\frac{8}{11}$ of what number?

4. 342 is $\frac{3}{7}$ of what number? **7.** 539 is $\frac{7}{12}$ of what number?

8. A boy sold 32 newspapers, which was $\frac{4}{5}$ of the number he had to sell. How many did he have at first?

9. The admissions to a fair, which were $\frac{15}{64}$ of the total receipts, amounted to $135. What were the total receipts?

10. One year 220,000 Italians arrived in the United States. If they were $\frac{11}{50}$ of all the immigrants, how many immigrants arrived that year?

REVIEW OF FRACTIONS

1. The sum of two fractions is $1\frac{7}{8}$. One fraction is $\frac{2}{3}$; what is the other?

2. The product of two fractions is $\frac{5}{8}$. One fraction is $1\frac{1}{5}$; what is the other?

3. If $\frac{3}{4}$ of an acre of land costs $45, how much will 5 acres cost?

4. A boy, after spending $\frac{3}{5}$ of his money, had $1.60 remaining. How much money had he at first?

5. If butter is selling at 28 cents a pound, how much will $3\frac{1}{2}$ pounds cost?

6. A man owned $\frac{3}{4}$ of a farm and sold $\frac{1}{3}$ of his share for $1250. At that rate how much was the whole farm worth?

7. If $1\frac{1}{2}$ acres yield 120 bushels of potatoes, how many bushels will 3 acres yield at the same rate?

SUGGESTION. — 3 are how many times $1\frac{1}{2}$?

8. Compare $\frac{1}{2}$ and $\frac{1}{4}$; $\frac{1}{3}$ and $\frac{1}{9}$; $\frac{1}{4}$ and $\frac{1}{12}$.

9. At $16\frac{2}{3}$ cents a pound, how many pounds of meat can be bought for $1? $2? $4? $5?

10. What part of 60 is 10? 15? 20? 40?

11. I sold a bicycle for $15; this was $\frac{3}{7}$ of what I paid for it. How much did I pay for it?

12. If 20 dozen eggs are worth $5.60, how much are 50 dozen worth?

13. If a telegram of 10 words costs 25 cents, how much will a telegram of 30 words cost at 2¢ for each extra word?

14. Compare 6 and 18; 8 and 48; 36 and 12.

15. How does a city lot worth $6000 compare with another worth $5000?

16. If James's age is 16 years and his father is $2\frac{1}{2}$ times as old, what is the difference in their ages? James's age is what part of his father's?

17. How much will 9 lemons cost at the rate of 40 cents a dozen? How much will $1\frac{1}{2}$ dozen cost? 3 dozen?

18. A field is 40 rods long. The width is $\frac{3}{4}$ of its length. What is the distance around the field?

19. A traveler walked $25\frac{1}{2}$ miles the first day, $30\frac{1}{4}$ miles the second day, $29\frac{3}{4}$ miles the third day, $27\frac{3}{8}$ miles the fourth day, and $25\frac{1}{8}$ miles the fifth day. How far did he travel in the 5 days, and what was his average rate per day?

20. An errand boy receives \$4 a week. In how many weeks can he earn enough to pay for a pair of shoes at $\$2\frac{1}{4}$, a suit at \$ 10, a hat at $\$1\frac{1}{2}$, and 3 shirts at $\$\frac{3}{4}$ each?

21. Divisor $25\frac{1}{2}$; quotient 200. Find the dividend.

22. Multiply the sum of $4\frac{1}{2}$ and 3 by 8.

23. At $\$1\frac{1}{4}$ a yard, how many yards of Brussels carpet can be bought for \$30?

24. A lady bought $12\frac{1}{2}$ yards of muslin at 8 cents a yard, $4\frac{1}{2}$ yards of ribbon at 4 cents a yard, 8 yards of dress goods at 75 cents a yard, and gave in payment a ten-dollar bill. How much change should she receive?

25. The sum of three numbers is 418; the first is $198\frac{3}{4}$, and the second $206\frac{1}{2}$. What is the third?

26. Mr. Jones sold $\frac{7}{8}$ of his farm for \$8890. At the same rate what was the value of the remainder?

27. Mr. White paid \$2700 for a three-eighths interest in a store. How much was the entire store worth?

28. A teacher spends \$1062.50 a year, which is $\frac{5}{8}$ of his salary. How much is his salary?

29. A dealer bought 960 bushels of wheat at 84 cents a bushel. He sold $\frac{1}{4}$ of it at 88 cents a bushel, $\frac{1}{3}$ of it at 87 cents a bushel, and the remainder at 89 cents a bushel. What was his entire gain?

30. Compare $\frac{2}{3}$ and $\frac{8}{9}$; $\frac{3}{4}$ and $\frac{5}{6}$; $\frac{2}{6}$ and $\frac{1}{4}$.

31. Thomas buys a necktie for $\$\frac{1}{2}$, a pair of gloves for $\$1\frac{2}{5}$, and 2 shirts at $\$\frac{3}{4}$ apiece. How much change should he receive from a five-dollar bill?

32. If a man's wages are $\$3\frac{1}{2}$ a day, and his daily expenses $\$1\frac{3}{4}$, how many weeks of 6 days each must he labor to save $\$105$?

33. Two men are 29 miles apart and travel toward each other, the one at the rate of $3\frac{1}{2}$ miles an hour and the other at $3\frac{3}{4}$ miles an hour. In how many hours will they meet?

34. Find the cost of $7\frac{1}{2}$ pounds of mackerel at 16 cents, $6\frac{1}{2}$ pounds of codfish at 12 cents, and 6 cans of salmon at the rate of 2 for a quarter.

35. What is the cost of 80 pounds of sugar at $6\frac{1}{4}¢$, $48\frac{1}{2}$ pounds of tea at $36¢$, and 124 pounds of coffee at $16¢$?

36. What is the cost of $2\frac{1}{2}$ pounds of coffee at $28¢$, $\frac{1}{2}$ pound of pepper at $24¢$, $3\frac{3}{4}$ pounds of butter at $28¢$, and $1\frac{1}{4}$ gallons of vinegar at $32¢$?

37. Mrs. Baker willed to her daughter $\frac{1}{3}$ of her estate, which amounted to $2400. To her son she left $\frac{1}{2}$ of her estate, and the rest to her sister. How much did the son and the sister each receive?

38. A man spent $\$22\frac{1}{2}$; then earned $18.75, and then had $50. How much had he at first?

39. What is the cost of 3 barrels of sugar weighing 300 pounds, 310 pounds, 312 pounds, respectively, at $5\frac{1}{2}¢$ a pound?

40. A farmer has 300 sheep. He sells $\frac{2}{5}$ of them at $\$3\frac{1}{2}$ each, and the remainder at $\$4$ each. How much does he get for all?

41. If 6 dozen oranges cost $\$2\frac{1}{10}$, how much will 24 dozen cost?

Find the amount of the following bills:

42.
$6\frac{1}{2}$ yd. of ribbon @ $10¢$.
$12\frac{3}{4}$ yd. of muslin @ $8¢$.
$11\frac{1}{2}$ yd. silk @ $98¢$.
$10\frac{1}{2}$ yd. lace @ $20¢$.

43.
$12\frac{1}{2}$ lb. of sugar @ $6¢$.
$2\frac{1}{4}$ lb. butter @ $28¢$.
$4\frac{3}{4}$ lb. of lard @ $16¢$.
$3\frac{1}{2}$ qt. beans @ $10¢$.

44. A blackboard is 21 feet long and $3\frac{1}{2}$ feet wide. The width is what part of the length?

45. I paid $\$4000$ for $2\frac{1}{2}$ acres of land, and after taking $\frac{3}{8}$ of an acre for streets divided the remainder into lots of $\frac{1}{16}$ of an acre each. I sold the lots at $\$150$ each. Find the gain.

46. I spent $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{5}$ of my money and had $\$20$ left. How much had I at first?

47. Compare 42 and 28. If 28 men earn $\$56$ in a day, how much will 42 men earn in the same time?

48. What is the distance around a lot $60\frac{1}{2}$ feet front and 121 feet deep?

49. How many times can I fill a pail holding $\frac{1}{2}$ a gallon from a 12-gallon tank that is full?

50. Frank is 16 years old, and $\frac{7}{8}$ of his age is $\frac{7}{10}$ of John's age. How old is John?

51. Find the product of $104\frac{3}{4} \times 24$.

52. Find the cost of the following articles: 12 yd. velvet @ $\$2.25$; $12\frac{1}{2}$ yd. suiting @ $50¢$; $7\frac{3}{4}$ yd. of dimity @ $20¢$.

53. A carpenter worked $4\frac{1}{2}$ days one week, $5\frac{1}{4}$ days the next week, and $5\frac{3}{4}$ days the next week. How many days did he work in the three weeks?

54. If he received \$3.50 a day, how much did he earn?

55. A merchant sold 20 pounds of sugar at $5\frac{1}{2}$ cents a pound, $12\frac{1}{2}$ pounds of tea at 36 cents a pound, $16\frac{1}{2}$ pounds of rice at 8 cents a pound, and $1\frac{1}{2}$ dozen oranges at 40 cents a dozen. Find the amount of the sale.

56. A dealer bought 80 bushels of apples at $\$\frac{3}{4}$ a bushel, 60 bushels at $\$\frac{2}{5}$ a bushel, and 40 bushels at $\$\frac{7}{10}$ a bushel. How much did the apples cost?

57. He retailed $\frac{1}{4}$ of them at \$1 a bushel, $\frac{1}{2}$ of them at $\$\frac{9}{10}$ a bushel, and the remainder at $\$\frac{4}{5}$ a bushel. How much did he gain?

58. From a piece of cloth a tailor cut 6 garments, each containing $4\frac{1}{2}$ yards, and there remained 4 yards. How many yards did the piece contain at first?

59. A grocer sold $1\frac{1}{2}$ pounds of butter to one customer, $2\frac{1}{4}$ pounds to another, $3\frac{3}{4}$ pounds to another, and $5\frac{1}{2}$ pounds to another. How much butter did he sell?

60. I have \$30 in gold, \$18 in silver, and the remainder in paper money. The gold is what part of the silver and the paper money?

61. A gardener raised 20 bushels of beans. He sold $\frac{2}{5}$ of them at \$1.80 per bushel and the remainder at \$1.65 per bushel. How much did he get for the crop?

62. A real estate dealer bought a plan of 12 lots for \$1800. He sold $\frac{1}{3}$ of them at \$200 each, $\frac{1}{2}$ of the remainder at \$180 each, and the remainder at \$160 each. How much did he gain?

DECIMALS

DECIMAL DIVISIONS OF A UNIT

Any unit may be divided into 10ths, 100ths, 1000ths, etc.

A **decimal fraction** is any number of tenths, hundredths, thousandths, etc., of a unit. When expressed with a decimal point, without a written denominator, it is usually called a **decimal.**

Thus $\frac{5}{10}$ and .5, $\frac{5}{100}$ and .05, $\frac{5}{1000}$ and .005 are *decimal fractions*, but the term *decimal* is usually restricted to the forms .5, .05, .005, etc. To write or express a number *decimally*, however, means to write it with the decimal point.

1. Express decimally: $\frac{5}{10}$, $\frac{25}{100}$, $\frac{35}{1000}$, $\frac{2}{10}$, $\frac{355}{1000}$.

2. In 5.55, the figure in tenths' place equals how many times the figure in hundredths' place? The figure in ones' place equals how many times the figure in tenths' place?

In any decimal or whole number 10 *units of any place* = 1 *unit of the next place to the left.*

The **decimal point** is used to separate the units and parts of units. It is always placed at the right of ones' place and before tenths' place.

A **mixed decimal** is an integer and a decimal united; as, 4.5.

3. What is the *first* place to the right of a decimal point called? the *second* place? the *third* place?

4. What is the largest decimal division of any unit? the *second* largest? the *third* largest?

NOTATION AND NUMERATION OF DECIMALS

Observe the table of places and the names of *integral* and *decimal* units.

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones	Decimal point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths
3,	7	4	5,	5	5	3	.	4	5	2	6	8	7

Name the decimal places, beginning at tenths. Read a decimal as though its denominator were expressed; thus, .045 is read *forty-five thousandths*.

Read:

1. .025	4. .5075	7. .5005
2. .4	5. .034	8. .0007
3. .0745	6. .2378	9. .2002

10. We read 2.005, 2 *and* 5 thousandths. What word stands for the decimal point? In reading a mixed decimal, what must we always call the decimal point?

NOTE. — Pupils should receive a thorough blackboard drill in reading decimals and mixed decimals. Special care must be taken not to read *and* except between ones and tenths.

Read:

11. 25.005	14. 20.075	17. 98.7405
12. 600.45	15. 5.4004	18. 400.0504
13. 800.5	16. 700.0006	19. 305.0209

Express decimally:

20. $\frac{4}{10}$	**24.** $600\frac{23}{10000}$	**28.** $72\frac{1}{1000}$
21. $3\frac{7}{100}$	**25.** $\frac{5}{1000}$	**29.** $87\frac{9}{1000}$
22. $\frac{405}{1000}$	**26.** $\frac{404}{10000}$	**30.** $\frac{6}{10000}$
23. $400\frac{2}{10}$	**27.** $5\frac{75}{100}$	**31.** $\frac{808}{1000}$

32. What does the last place in enumerating the decimal always tell? What is the decimal unit in .5? in .05? in .005?

33. Five thousandths calls for how many decimal places? 5 must be written in what decimal place? How many naughts must be prefixed to 5?

34. Tenths means how many decimal places? hundredths? thousandths? ten-thousandths? hundred-thousandths? millionths?

In writing a decimal, write the last figure of the decimal in the place called for. If the significant figures do not occupy all the decimal places, prefix the necessary naughts.

35. Express decimally five tenths; five hundredths; five thousandths; six and four tenths; ten and twenty-four thousandths; sixty-five ten-thousandths; eight and eight thousandths: six hundred and four ten-thousandths.

36. Five millionths; twenty-five hundred thousandths; six and seven thousandths; eight hundred and five tenths.

Equivalents of common and decimal fractions.

1. $\frac{1}{2} = \frac{?}{10}$; $\frac{1}{2} = \frac{?}{100}$; $\frac{1}{4} = \frac{?}{100}$; $\frac{3}{4} = \frac{?}{100}$?

2. Since $\frac{1}{8} = \frac{12\frac{1}{2}}{100}$; $\frac{3}{8} = \frac{?}{100}$; $\frac{5}{8} = \frac{?}{100}$?

3. $\frac{5}{6} = \frac{?}{100}$; $\frac{1}{3} = \frac{?}{100}$; $\frac{2}{3} = \frac{?}{100}$?

4. $\frac{5}{12} = \frac{?}{100}$; $\frac{7}{12} = \frac{?}{100}$; $\frac{3}{8} = \frac{?}{100}$?

5. Give the decimal equivalents for: $\frac{1}{2}$, $\frac{3}{4}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{5}{12}$, $\frac{7}{12}$, $\frac{3}{8}$.

ADDITION AND SUBTRACTION OF DECIMALS

1. Why must $\frac{3}{4}$ and $\frac{2}{3}$ be changed to twelfths before they can be added or subtracted?

2. What kind of fractional units, then, can be added or subtracted?

3. Observe that decimals are only so many 10ths, 100ths, 1000ths, etc., of a unit expressed by placing a period before the numerator and omitting the denominator. In adding or subtracting decimals, then, the decimal must be written so that like units are under each other.

Written Work

1. Add .75 + .055 + .096

.75	Test: .7 + 0 + 0	= .7
.055	.05 + .05 + .09	= .19
.096	.006 + .005 + .00	= .011
.901		.901

2. From 15 take 6.387.

15 may be written 15.000.

15.000	Do naughts annexed to an integer change its value? Do naughts annexed to a decimal change its value?
6.387	
8.613	

Add:

3. 25,005, .75, .005, .72, 7., 8.445, .875, .05, .0745, .6475.

4. Subtract 75.005 from 485.007.

5. Subtract .8075 from 23.

6. Subtract .5075 from 23.004.

7. Add 2.5, 11.25, 18.042, 27.0548.

8. Add 1.45, 3.06, 6.605, .09.

9. Add 4.24, 8.2, 6.006, 19.098.

10. Add 11.01, 3.7, 10.01, 2.005.

11. Add .7, .4285, 18.054, 8.0108.

12. Add .002, 22.5607, 1.114, 18.

13. Add 126, 2578, 9.009, .00101, 2.02, .0245.

14. Add .0402, 48.0148, .07089, .1607, 17.0017.

15. Add 89.4004, 75.8002, 761.0612, 1245.0005.

Find differences:

16. 5.32 − 3.245

17. 10.004 − 6.205

18. 125.04 − 86.008

19. 12 − 3.001

20. 350.25 − 180.175

21. 221.201 − 175.1254

22. 434.5196 − 178.3021

23. 90.909 − 9.9009

24. 18 − 11.006

25. 245.045 − 138.1256

26. 100.101 − 95.095

27. 300.333 − 195.033

28. 20.93875 − 15.55

29. 100 − .9999

30. From .06 + .0875 + 49.03 take .025 + 2.0025 + 43.701.

31. From the sum of .2305 + .9105 take 1.

32. From the sum of 27.045 and .7001 take their difference.

33. Add as indicated and test by totals:

$$\begin{array}{rrrrl} .075 + & 6.875 + & .901 + & 10.101 & = \text{——} \\ 6.375 + & .057 + & .057 + & 9.704 & = \text{——} \\ .598 + & 2.079 + & .864 + & 12.006 & = \text{——} \\ .803 + & .868 + & 9.805 + & .011 & = \text{——} \\ \underline{9.603} + & \underline{8.789} + & \underline{7.504} + & \underline{.023} & = \text{——} \\ \text{Totals:} \quad + & + & + & & = \text{——} \end{array}$$

34. How many yards equal 5 pieces of cloth containing 25.25 yd., 32.625 yd., 40.81 yd., 45.5 yd., and 48.75 yd. respectively?

35. A clerk's income for one year is $600. He spends $487.75. How much does he save?

36. The sum of two numbers is 118.6, and one of the numbers is 14.247. What is the other number?

37. What is the weight in tons of 4 loads of coal, which weigh respectively 1.5 T., 1.1875 T., 1.3125 T., and 1.9 T.?

38. If a tailor uses 4.375 yards of cloth in making a suit, how much remains of a piece of cloth containing 12.25 yards?

39. Find the cost of three tables at $9.50, $12.75, and $15.80.

40. A meter contains 39.37 inches. How much larger is a meter than a yard?

41. How many feet of lumber are there in three piles measuring 2675.25 ft., 6785.875 ft., and 5674.5 ft.?

42. A farmer bought 75.5 pounds of clover seed and sowed 54.25 pounds. How much had he remaining?

MULTIPLICATION OF DECIMALS

Multiplying a decimal by an integer or by a decimal.

1. How many are $5 \times .3$? $5 \times .03$? $5 \times .003$?

2. Multiply $\frac{3}{10}$ by $\frac{5}{10}$ and express the product decimally.

$\frac{3}{10} \times \frac{5}{10} = \frac{15}{100} = .15$. Hence, $.3 \times .5 = .15$.

3. Multiply $\frac{3}{10}$ by $\frac{5}{100}$ and express the product decimally.

$\frac{3}{10} \times \frac{5}{100} = \frac{15}{1000} = .015$. Hence, $.3 \times .05 = .015$.

In the above examples compare the number of decimal places in the multiplicand and multiplier together with the number of decimal places in the product.

Written Work

1. Multiply 5.8 by 6.

$$\begin{array}{r} 5.8 \\ 6 \\ \hline 34.8 \end{array}$$

$6 \times .8 = 48$ tenths or 4 ones and 8 tenths. Write the .8 and carry 4. $6 \times 5 = 30$; $30 + 4 = 34$. Hence, $6 \times 5.8 = 34.8$

2. Multiply .25 by .13.

$$\begin{array}{r} .25 \\ .13 \\ \hline 75 \\ 25 \\ \hline .0325 \end{array}$$

Study of Problem

a. What is the sum of the decimal places in the multiplier and multiplicand? in the product?

b. The product, then, must contain how many places?

c. How many significant figures are there in the product?

The* number of decimal places in the product of the two numbers *is the sum of the decimal places in both multiplier and multiplicand.

Multiply:

3.	4×5.6	**17.**	15×3.04	**31.**	99×234.17
4.	5×7.5	**18.**	8×10.34	**32.**	402×4.022
5.	3×8.4	**19.**	$18 \times .004$	**33.**	$472 \times .0504$
6.	7×9.3	**20.**	$122 \times .024$	**34.**	$122 \times .5625$
7.	8×6.8	**21.**	$215 \times .015$	**35.**	$144 \times .00321$
8.	6×5.8	**22.**	$83 \times .007$	**36.**	96×1.0208
9.	5×7.2	**23.**	212×2.042	**37.**	407×4.003
10.	7×6.5	**24.**	$432 \times .078$	**38.**	$.04 \times .078$
11.	8×5.7	**25.**	$101 \times .012$	**39.**	$.64 \times .016$
12.	$.12 \times .25$	**26.**	14×89.76	**40.**	$.012 \times .024$
13.	$.22 \times .14$	**27.**	$112 \times .092$	**41.**	$.625 \times .001$
14.	$.25 \times .25$	**28.**	$363 \times .003$	**42.**	$.872 \times .096$
15.	$.32 \times .43$	**29.**	90×5.78	**43.**	$.0004 \times .004$
16.	$.41 \times .55$	**30.**	36×6.48	**44.**	$.0505 \times .55$

45. $.145 \times .625$	52. $.325 \times .125$	59. $.0216 \times .027$
46. $.046 \times .752$	53. $.043 \times .057$	60. $.0244 \times .014$
47. $.125 \times .246$	54. $.016 \times .235$	61. $.009 \times .099$
48. $.414 \times .601$	55. $.534 \times .223$	62. $.1101 \times .101$
49. $.851 \times .004$	56. $.261 \times .175$	63. $.3756 \times .124$
50. $.654 \times 1.08$	57. $.022 \times .022$	64. $.0456 \times .032$
51. $.506 \times 24.6$	58. $.632 \times .085$	65. $.0038 \times .097$

66. If a boy receives \$1.25 a day, how much will he receive in 36 days?

67. How many acres are there in three fields of 65.875 acres each?

68. There are 31.5 gallons in a barrel. How many gallons are there in 63 barrels?

69. A merchant sold 75 yards of muslin at \$.125 per yard. How much did he receive for it?

70. Find the cost of 15.3125 acres of land @ \$50.

71. Find the product of .095 multiplied by 3.17.

72. Find the product of 909 multiplied by 9.99.

73. Find the product of .0725 multiplied by 480.

74. If a gallon of water weighs 8.338 pounds, find the weight of 24 gallons.

75. A franc equals \$.193. How many cents are there in 125 francs?

76. A meter equals 1.093 yards. Find the number of yards in 64 meters.

77. 12.26	78. .675	79. 20.25
4.08	.54	.079
9808	2700	18225
4904	3375	14175
500.208	.36450	1.59975

Find products:

80. 2.5×1.75	88. 8.132×2.4	96. 44.006×6.044
81. 14.4×1.6325	89. 10.001×7.07	97. 5.117×1.88
82. 10.85×2.975	90. 52.5×7.08	98. 16.004×16.64
83. 60.95×3.03	91. 57.135×8.56	99. 32.406×15.108
84. 15.02×5.001	92. 9.901×1.99	100. 41.041×9.009
85. $40.0023 \times .021$	93. 456.375×44.8	101. 56.561×6.01
86. 35.007×4.8	94. 12.063×14.204	102. 3.0054×1.0405
87. 1.78×1.024	95. 101.1×11.01	103. 5.00237×3.0701

104. If a man walks 4.75 miles in an hour, how far will he walk in 11.25 hours?

105. How many yards of cloth are there in 12.4 bales, each bale containing 56.375 yards?

106. A farmer averages 35.875 bushels of wheat from 12.95 acres. How many bushels does he harvest?

107. Find the cost of 18.75 dozen eggs at $.125 a dozen.

Multiplying by moving the decimal point.

1. Multiply 5.25 by 10; by 100; by 1000.

$10 \times 5.25 = 52.50$
$100 \times 5.25 = 525.00$
$1000 \times 5.25 = 5250.00$

Study of Problem

a. How may we multiply a decimal by 10? by 100? by 1000?

b. How is the value of a number affected by moving the decimal point *one place* to the right? *two places? three places?*

c. How, then, may a number be multiplied by 10? by 100?

Multiply first by 10; then by 100; then by 1000.

2. 42.07	5. 16.94	8. 222.461	11. .005
3. 113.55	6. 849.02	9. 333.059	12. 4.009
4. 264.03	7. 500.09	10. 29.004	13. 13.655

DIVISION OF DECIMALS

Dividing a decimal or a mixed decimal by an integer.

1. Divide 48 hundredths by 4; by 6; by 8; by 12.

2. $.48 \div 4 = ?$ $.48 \div 6 = ?$ $.48 \div 12 = ?$

3. Compare $6)\overline{.72}$ $.12$ and $.72 \div 6$.

4. Divide 24 and 64 hundredths by 4; by 8.

5. Divide thus: $4)\overline{24.64}$ 6.16 $\quad 8)\overline{24.64}$ 3.08 $\quad 12)\overline{48.72}$ 4.06

In each of the above problems the decimal or mixed decimal was simply separated or *partitioned* into equal parts.

Written Work

Find quotients:

1. $.64 \div 8$	5. $108.09 \div 9$	9. $54.06 \div 6$
2. $.72 \div 6$	6. $96.08 \div 8$	10. $48.72 \div 12$
3. $8.04 \div 2$	7. $84.07 \div 7$	11. $50.05 \div 5$
4. $72.06 \div 6$	8. $64.24 \div 8$	12. $36.96 \div 12$

13. Divide .1275 by 25.

$$25)\overline{.1275} \quad .0051 \quad 125 \quad 25 \quad 25$$

14. Divide 192.96 by 16.

$$16)\overline{192.96} \quad 12.06 \quad 16 \quad 32 \quad 32 \quad 96 \quad 96$$

Divide as in integers, placing the decimal point in the quotient directly above or below the point in the dividend, before you begin to divide.

Divide:

15. .25 ÷ 5	23. .027 ÷ 18	31. .4125 ÷ 35
16. .64 ÷ 16	24. .444 ÷ 50	32. .2286 ÷ 127
17. .02 ÷ 40	25. .125 ÷ 50	33. .0124 ÷ 20
18. .75 ÷ 60	26. .966 ÷ 46	34. .5058 ÷ 18
19. .49 ÷ 140	27. .018 ÷ 12	35. .1728 ÷ 24
20. .01 ÷ 100	28. .546 ÷ 21	36. .0001 ÷ 1000
21. .05 ÷ 500	29. .003 ÷ 10	37. .0343 ÷ 14
22. .03 ÷ 100	30. .868 ÷ 16	38. .5184 ÷ 96

39. If 8 yards of muslin are sold for $.80, what is the price per yard?

40. When 144 pens are sold for $.72, what is the price per pen?

41. If 10 sheets of paper are sold for $.05, what is the price per sheet?

42. If $.27 is paid for 1000 cubic feet of gas, what is the price per cubic foot?

43. Find the sum of the quotients:

.02 ÷ 2 =	.035 ÷ 14 =
.034 ÷ 17 =	.368 ÷ 16 =
.06 ÷ 60 =	.18 ÷ 12 =
.024 ÷ 15 =	.045 ÷ 18 =
.168 ÷ 7 =	.25 ÷ 125 =
.0044 ÷ 11 =	.48 ÷ 28 =

Divide:

44. 1.625 ÷ 25	49. 2.07 ÷ 46	54. 2.7355 ÷ 35
45. 24.36 ÷ 12	50. 31.2 ÷ 36	55. 31.288 ÷ 48
46. 172.8 ÷ 24	51. 2.31 ÷ 55	56. 137.95 ÷ 31
47. 14.76 ÷ 41	52. 1.17 ÷ 65	57. 106.32 ÷ 24
48. 1.105 ÷ 65	53. 16.5 ÷ 22	58. 17.172 ÷ 53

Dividing any number by a decimal.

Divide 5.5 by 5.5.

1. $5.5 \div 5.5 = 1$
   $55 \div 55 = 1$

Study of Problem

a. What is the first quotient? the second?

b. What was done to the first problem to make the second?

c. Did moving the decimal point to the right, in both dividend and divisor, change the quotient?

Multiplying both dividend and divisor by the same number does not change the quotient.

Observe that: $.6\overline{).66} = 6\overline{)6.6}$; $.02\overline{).004} = 2\overline{).4}$

2. Divide 6.25 by .5.

$6.25 \div .5 = 62.5 \div 5$

$5\overline{)62.5}$
12.5

a. Make the divisor an integer by moving the decimal point one place to the right in both dividend and divisor.

b. Show that this does not affect the quotient.

c. Solve, placing the decimal point in the quotient before beginning to divide.

3. Divide 62.5 by .025.

$62.5 \div .025 = 62500 \div 25.$

$25\overline{)62500}$
2500

a. Make the divisor an integer by moving the decimal point three places to the right.

b. Solve, placing the decimal point in the quotient before beginning to divide.

4. Divide 25 by .004

$25 \div .004 = 4\overline{)25000}$ $\quad 6250$

Move the decimal point the required number of places to the right in both dividend and divisor so that the divisor will be a whole number. Divide as in the division of a decimal by an integer.

The use of the caret in division of decimals.

Another method is to mark off by a **caret** (∧) as many decimal places from the right of the decimal point in the dividend as there are decimal places in the divisor, and then to divide as in integers, placing the decimal point directly below or above the caret in the dividend. Thus,

$$.5\overline{).005} = .5\overline{).0\wedge05}$$

It is evident in the above problem that if both the dividend and the divisor were changed so as to make the divisor a whole number, the decimal point in the dividend would be in the place occupied by the caret. The decimal point must therefore be placed in the quotient directly above or below the caret.

The caret determines the *position* of the decimal point in the quotient and at the same time retains the *identity* of the problem.

Written Work

1. Divide 673.5 by .25.

$$\begin{array}{r} 2694 \\ .25\overline{)673.50\wedge} \\ 50 \\ \hline 173 \\ 150 \\ \hline 235 \\ 225 \\ \hline 100 \\ 100 \\ \hline \end{array}$$

NOTE. — If, as in example 1, no figure in the dividend is of the same order as the right-hand figure of the divisor, one or more naughts must be annexed to the dividend before writing the caret.

2. Divide 6.75 by .05.

$$\begin{array}{r} .05\overline{)6.75\wedge} \\ 135. \end{array}$$

Mark off by a **caret** *the same number of decimal places from the right of the decimal point in the dividend as there are decimal places in the divisor. Divide as in integers, placing the decimal point in the quotient directly above or below the caret, before beginning to divide.*

Divide in the most convenient way:

3. 6 by .3	11. 70 by .0056	19. 1 by .001
4. 9 by .06	12. 154 by .28	20. 10 by .01
5. 21 by .7	13. 78 by .052	21. 17 by .68
6. 10 by .01	14. 190 by .0076	22. 112 by .032
7. 25 by .125	15. 115 by 6.25	23. 324 by .27
8. 80 by .3125	16. 18 by .9375	24. 1904 by .119
9. 86 by .75	17. 4 by .016	25. 114 by .76
10. 128 by .032	18. 48 by .1875	26. 896 by .0256

27. If the rainfall in a certain place is, on an average, .01 of an inch a day, in how many days does the rainfall amount to 3 inches?

28. How many layers of gold leaf will be required to form a tablet 5 inches thick, if each layer is .001 of an inch thick?

29. If one yard of flannel is worth $.625, how many yards can be bought for $575?

30. How many pencils can be bought for $324 at $.0075 each?

31. When slate pencils are worth $.0055 apiece, how many can be purchased for $22?

Find quotients and prove results:

32. .04 ÷ .002	40. .004 ÷ .004	48. .728 ÷ .13
33. .8 ÷ .25	41. .1952 ÷ 16	49. .3136 ÷ .224
34. .125 ÷ .5	42. .00624 ÷ .8	50. .4725 ÷ .2
35. .112 ÷ 7	43. .0247 ÷ .019	51. .4375 ÷ .125
36. .036 ÷ 4	44. .8799 ÷ .7	52. .17225 ÷ 1325
37. .0001 ÷ .01	45. .08799 ÷ .007	53. .7665 ÷ .365
38. .0187 ÷ .011	46. .15158 ÷ .286	54. .2944 ÷ .512
39. .555 ÷ .37	47. .408375 ÷ .135	55. .421875 ÷ .1125

Find the sum of the quotients:

56.	$.02 \div .04 =$	**57.**	$.49 \div .07 =$
	$.05 \div .4 =$		$.016 \div .04 =$
	$.001 \div .01 =$		$.6 \div .8 =$
	$.25 \div .025 =$		$.216 \div .18 =$
	$.75 \div .125 =$		$.128 \div .16 =$
	$.044 \div .08 =$		$.03 \div .003 =$
	$.056 \div .14 =$ _____		$.045 \div .15 =$ _____

58. At $.20 a pound, how many pounds of coffee can be bought for $.85?

59. When potatoes are selling at $.125 a peck, how many pecks can be bought for $.75?

60. A mark is $.238. How many marks equal $952?

61. At $.025 each, how many pens can be bought for $.35?

Find quotients:

62.	$.655 \div .0131$	**69.**	$.3 \div .03$	**76.**	$1.111 \div 11.11$
63.	$.75 \div .0125$	**70.**	$1.5 \div .005$	**77.**	$100.5 \div 1.005$
64.	$.3625 \div .125$	**71.**	$10.8 \div .12$	**78.**	$8.686 \div 86.86$
65.	$1.44 \div .036$	**72.**	$1.32 \div 11$	**79.**	$.01 \div .001$
66.	$.9 \div .015$	**73.**	$31.75 \div .025$	**80.**	$100 \div 1000$
67.	$.1 \div 1.25$	**74.**	$.5475 \div 1.5$	**81.**	$7.25 \div .025$
68.	$10 \div 2.25$	**75.**	$1.728 \div 17.28$	**82.**	$1.225 \div 3.5$

REDUCTION OF DECIMALS

Reducing a decimal to a common fraction.

1. Express .5 with the denominator tenths; thus, $\frac{5}{10}$; then reduce this fraction to its lowest terms.

2. Express in the form of fractions reduced to their lowest terms: .25; .20; .50; .75; .80.

Written Work

1. Change $.87\frac{1}{2}$ to a common fraction in its lowest terms.

$.87\frac{1}{2} = .875 = \frac{875}{1000} = \frac{7}{8}$

We express the denominator of the decimal and reduce the resulting fraction to its lowest terms.

2. Change $.66\frac{2}{3}$ to a common fraction.

$.66\frac{2}{3} = \frac{200}{3} \div 100 = \frac{200}{300} = \frac{2}{3}$

Since $66\frac{2}{3} = \frac{200}{3}$, $.66\frac{2}{3} = \frac{200}{3} \div 100$, or $\frac{200}{300}$. This reduced to its lowest terms equals $\frac{2}{3}$.

3. Change .075 to a fraction in its lowest terms.

$.075 = \frac{75}{1000} = \frac{3}{40}$

We express .075 with its denominator 1000 and reduce the fraction to its lowest terms, $\frac{3}{40}$.

4. Change the following decimals to fractions in their lowest terms:

.35	.125	$.16\frac{2}{3}$	$.83\frac{1}{3}$
.24	.375	$.41\frac{2}{3}$	$.33\frac{1}{3}$
.205	.0075	$.62\frac{1}{2}$	$.06\frac{1}{4}$

Changing a common fraction to a decimal.

1. How can you change $\frac{1}{2}$ to tenths? Express this result decimally.

2. Change $\frac{1}{4}$ to hundredths and express the result decimally.

3. Change to hundredths expressed decimally: $\frac{1}{5}$; $\frac{2}{5}$; $\frac{3}{4}$; $\frac{3}{5}$; $\frac{3}{10}$.

4. Is there any difference in value between $\frac{3}{4}$ and $3 \div 4$? between $\frac{5}{8}$ and $5 \div 8$?

Written Work

1. Change $\frac{3}{4}$ to a decimal.

$\frac{3}{4} = 3 \div 4 = 4\overline{)3.00}$ $\quad 0.75$

Since a fraction may be regarded as an expression of division, $\frac{3}{4} = 3 \div 4$. Annexing naughts and dividing as on p. 195, we find the answer 0.75. Proof: $0.75 = \frac{75}{100} = \frac{3}{4}$.

NOTE.—A decimal point must be placed after an integer before naughts can be annexed.

2. Change $\frac{5}{16}$ to a decimal.

$\frac{5}{16} = 5 \div 16 = 16\overline{)5.0000}$ $\quad 0.3125$

Proceeding as in example 1, we annex four naughts and divide. The result is 0.3125.

***A* common fraction is changed to an equivalent decimal** *by placing a decimal point after ones' place in the numerator and dividing by the denominator.*

Change to equivalent decimals and prove:

3. $\frac{1}{5}$	6. $\frac{7}{8}$	9. $\frac{7}{20}$	12. $\frac{11}{16}$
4. $\frac{4}{5}$	7. $\frac{5}{8}$	10. $\frac{6}{25}$	13. $\frac{13}{25}$
5. $\frac{3}{8}$	8. $\frac{3}{10}$	11. $\frac{9}{16}$	14. $\frac{11}{20}$

In changing $\frac{4}{9}$ to a decimal, thus, $9\overline{)4.000}$ $\quad 0.444\frac{4}{9}$, it is evident that the divisor is not contained in the dividend an integral number of times. The quotient may be indicated as above, or a + sign may take the place of the fraction to show an undivided remainder. Thus, $9\overline{)4.000}$ $\quad 0.444+$.

Change to equivalent decimals:

15. $\frac{5}{9}$	18. $\frac{7}{12}$	21. $\frac{11}{17}$	24. $\frac{8}{9}$
16. $\frac{4}{11}$	19. $\frac{8}{18}$	22. $\frac{8}{14}$	25. $\frac{7}{18}$
17. $\frac{6}{7}$	20. $\frac{7}{15}$	23. $\frac{5}{7}$	26. $\frac{8}{15}$

Change to mixed decimals:

Thus, in example 27, $\frac{3}{4} = .75$, therefore $16\frac{3}{4} = 16.75$.

27. $16\frac{3}{4}$	29. $24\frac{5}{16}$	31. $4.17\frac{4}{15}$	33. $25\frac{5}{12}$
28. $12\frac{5}{8}$	30. $11\frac{16}{25}$	32. $12\frac{5}{7}$	34. $18\frac{9}{14}$

REVIEW OF FRACTIONS AND DECIMALS

1. How many books at \$2.50 each can be bought for \$37.50?

2. Find the value of $.06 \times .03$ divided by 2.5.

3. How many farms, of $80\frac{3}{4}$ acres each, can be laid out from a tract of land containing 1211.25 acres?

4. How many yards of carpet, at \$1.15 a yard, can be bought for \$29.325?

5. Glycerin is 1.265 times as heavy as water. How many pounds of glycerin will equal in weight 4.6875 pounds of water?

Find the sum of the quotients:

6.		7.	
	$4.65 \div 1.55$		$31.906 \div 1.06$
	$12.5 \div 6.25$		$40.804 \div 1.01$
	$9.03 \div 3.01$		$3.861 \div 3.51$
	$11.75 \div 2.35$		$6.012 \div 5.01$
	$25.75 \div 5.15$		$36.542 \div 33.22$

8. If a man earns $\$1\frac{3}{4}$ in a day, how much will he earn in 34 days?

9. Find the value of $(3\frac{1}{2} \times 4\frac{2}{3}) \div 1\frac{3}{4}$.

10. If 7.375 yards of cloth cost \$29.50, how much will 10.875 yards cost?

11. Change $\frac{1}{2}$, $\frac{3}{4}$, $\frac{7}{8}$, $\frac{9}{16}$ to decimals.

12. Two men bought a store for \$6000; one paying .375 of it, and the other .625 of it. How much did each pay?

13. Read 100.01256.

14. If a boy earns \$1.25 a day, and a man earns \$3.25 a day, how long will it take the boy to earn as much as the man can earn in 15 days?

15. Multiply .0014 by 3.012.

16. What is the cost of 4879 feet of boards at \$.02 a board foot?

17. At \$.125 a pound, how many pounds of meat can be bought for \$9?

18. The product of two numbers is .0006. If one of the numbers is .04, what is the other?

19. A grocer bought 75 barrels of flour at \$4.75 a barrel. He sold 31 barrels at \$5.37 a barrel and the remainder at \$4.87 a barrel. Find his gain.

20. Explain the effect of removing the naught in the following: .015, .250, .018, .240, .016.

21. If 14.5 acres produce 168.635 bushels of wheat, what is the yield per acre?

22. Divide $\frac{14}{25}$ by .014.

23. My wheat crop was 272 bushels. I sold .625 of it at \$1.085 a bushel. How much did I get for it?

24. Multiply 1000 by .001, and divide the product by .01.

25. There are 31.5 gallons in a barrel. How many barrels are there in 2378.25 gallons?

26. A farm of $171\frac{7}{8}$ acres was sold in plots of 1.25 acres each, the price for each plot being \$75.50. How much was received for the farm?

27. Change $12\frac{3}{4}$ to a mixed decimal.

28. A cubic foot of pure water weighs $62\frac{1}{2}$ lb. Find the weight of 10.75 cu. ft.

29. Change .625 to a common fraction.

30. A teacher paid .16 of his salary for rent, .24 for provisions, .18 for clothes and other expenses, and put the remainder, $655.20, in the bank. How much was his salary?

31. Multiply 3.18 by 5.5, and write the result in words.

32. How many pounds of sugar can be bought for $16.25, if 5.5 pounds cost $.3575?

33. How many acres are there in a field containing .09 of 108.3125 acres?

34. How many miles will a freight train travel in $5\frac{5}{8}$ hours if it travels $20\frac{3}{4}$ miles an hour?

35. Divide forty-two ten-thousandths by five hundred twenty-five thousandths.

36. What is the value of $(1.5 + 3.75) \div .25$?

Find the cost of:

37. 90 lb. meal @ $3\frac{1}{3}$¢.

38. 256 lb. lard @ $12\frac{1}{2}$¢.

39. 186 doz. eggs @ $16\frac{2}{3}$¢.

40. 128 yd. gingham @ $6\frac{1}{4}$¢.

41. 120 yd. flannel @ 75¢.

42. 288 yd. linen @ $37\frac{1}{2}$¢.

43. 68 lb. butter @ 25¢.

44. 315 yd. prints @ $33\frac{1}{3}$¢.

45. How many yards of muslin, at $12\frac{1}{2}$ cents a yard, can be bought for $6?

46. At 75 cents a bushel, how many bushels of potatoes can be bought for $36?

47. At $16\frac{2}{3}$ cents a gallon, how many gallons of oil can be bought for $48?

48. How many yards of lining, at $6\frac{1}{4}$ cents a yard, can be bought for $4?

49. How many dozen eggs, at $12\frac{1}{2}$ cents a dozen, can be bought for $28?

50. At $16\frac{2}{3}$¢ a pound, how many pounds of coffee can be bought for $15?

SIMPLE ACCOUNTS FOR BOYS AND GIRLS

An **account** is a statement of the receipts and disbursements of any person.

There are two sides to an account: the *first*, or **debit side**, on which are entered all *receipts;* the *second*, or **credit side**, on which are entered all *disbursements*, or *amounts paid out.*

The person who owes the money is the **debtor** (Dr.), and the person to whom the money is due the **creditor** (Cr.).

The **balance** is the difference between the *debit* and *credit* sides.

SEPTEMBER 1, 1906.

		Dr.	Cr.
Sept. 1	Cash on hand	$13 10	
Sept. 1	Note-book, $.15; pencil, $.05		$ 20
Sept. 4	Arithmetic		50
Sept. 5	Geography		1 00
Sept. 7	Copy-book, $.10; ink and pens, $.08		18
Sept. 15	History		1 00
Sept. 17	Concert ticket		1 00
Sept. 25	Car fare		50
Sept. 29	Tools		60
	Balance, Cash on hand		8 12
		$13 10	$13 10

Continue the *balance* of each month through the following months to September, 1906.

NOTE TO PARENTS. — Children should be encouraged to keep their own personal accounts.

1. *October.* Oct. 3, Bought 1 pair of shoes, $2.50. 1 hat, $1.50. Oct. 8, Repairs to bicycle, $.75. Oct. 15, Earned $1.50. Oct. 17, Worked for Mr. Black and received $.75. Oct. 25, Saturday outing, $.60.

2. *November.* Nov. 5, Bought a sled, $.95. Nov. 9, Bought a cap, $.75. Nov. 15, Shoveled snow off Mrs. Graham's walk, $.30. Nov. 17, Sawed kindling wood for Mr. Goff, $.50. Nov. 26, Bought a knife, $.25. Nov. 30, Ran errands, $.35.

3. *December.* Dec. 3, Bought 1 pair of skates, $.75. Dec. 10, Received from Mr. Black for work in store, $1.00. Dec. 17, Expense for school supplies, $.17. Dec. 21, Received from Mrs. Williams for carrying in load of coal, $.30. Dec. 22, Bought Christmas presents, $3.75. Dec. 25, Christmas gift from Uncle James, $1.00. Dec. 29, Expense for having skates sharpened, $.10.

4. *January*, 1907. Jan. 5, Received from Mrs. Jones for fixing doorbell, $.15. Jan. 8, Bought 1 pair mittens, $.50. Jan. 15, Delivered bills around town for Mr. Black, $.50. Jan. 25, Bought necktie, $.25. Jan. 30, Bought "History of French Revolution," $.75.

5. *February.* Feb. 6, Worked on Saturday for Mr. Black, $.75. Feb. 11, Shoveled snow from sidewalk for Mr. Hart, $.25. Feb. 16, Ran errands, $.40. Feb. 20, Helped unload car of feed, $1.00. Feb. 26, Copied 2 leases for Mr. Irwin, $.75. Feb. 28, Bought pair of gloves, $1.25.

6. *March.* March 1, Cleaned yard for Mrs. Williams, $.50. March 6, Bought 2 pairs of socks, $.30. March 11, Bought new umbrella for mother, $1.75. March 15, Repaired fence for Mr. Jones, $.25. March 27, Car fare, $.30. March 30, Sold my old bicycle for $5.00.

7. *April.* Apr. 1, Burned paper and refuse for Mr. Hart, $.25. Apr. 8, Made garden for Mrs. Black, $.50. Apr. 10, Whitewashed cellar for Mrs. Goff, $.35. Apr. 15, Wheeled load of coal for Mr. Brown, $.35. Apr. 25, Bought 4 collars and 2 pairs of cuffs, $.90. Apr. 30, Bought necktie, $.25.

8. *May.* May 3, Bought straw hat, $1.00. May 7, Mowed lawn for Mrs. Jones, $.25. May 13, Repaired Mr. Brown's sidewalk, $.40; May 30, Bought baseball, $.50. May 31, Received a reward of $5.00 for finding a pocketbook containing $50, which I returned to owner.

9. *June.* June 1, Made $.20 selling papers. June 6, Worked a day for Mr. Black, $.75. June 10, Delivered package, $.25. June 17, Bought ball bat, $.50. June 20, Wheeled a trunk for Mr. Hart, $.25. June 29, Bought 1 pair of baseball shoes, $1.00.

10. *July.* July 4, Fireworks, $.50. July 6, Received from Mr. Black salary for week, $5.00. July 12, Bought 2 shirts, $1.50. July 13, Received week's salary, $5.00. July 15, Bought outing suit, $6.50. July 20, Received my salary, $5.00. July 25, Expense for small articles, $.95. July 27, Received my week's salary, $5.00. July 30, Received for overtime, for month, $7.50.

11. *August.* Aug. 3, Salary, $5.00. Aug. 8, Bought 1 pair of tan shoes, $2.50. Aug. 10, Received salary, $5.00. Aug. 15, Bought fishing tackle, etc., $3.75. Aug. 17, Received week's salary, $5.00. Aug. 31, Expenses for 2 weeks' vacation, $15.75.

Sept. 1, Balance, Cash on hand, ——.

Make out a statement at close of year, showing total receipts and disbursements, and proving final balance.

DENOMINATE NUMBERS

1. Write from memory the following tables:

Liquid Measures, Dry Measures, Avoirdupois Weight, Time Measures, and Measures of Length or Distance.

2. 1 yr. = —— mo. = —— da. = —— hr. = —— min. = —— sec.

3. 1 mi. = —— rd. = —— yd. = —— ft. = —— in.

4. 1 T. = —— cwt. = —— lb. = —— oz.

5. 1 bu. = —— pk. = —— qt.

6. 1 gal. = —— qt. = —— pt.

The standard or principal units of measure are as follows:

Liquid — **gallon.** Length or distance — **yard.**

Dry — **bushel.** Avoirdupois — **pound (16 oz.).**

Time — **day.**

All other measures are determined from the above unit measures. Thus, the ton is 2000 times 1 pound (16 oz.). The hour is $\frac{1}{24}$ of the day, the period of one revolution of the earth on its axis.

A **denominate number** is a concrete number whose unit is a measure established by custom or law; as, 10 feet or 5 pounds, in which 1 foot and 1 pound are the units of measure.

A **simple denominate** number is a number of one denomination; as, 12 rods, 2 ounces, 5 days, etc.

A **compound denominate** number is composed of two or more concrete numbers that express one quantity; as, 6 yards, 2 feet, and 4 inches. Here yards, feet, and inches are used to express but one quantity, length.

REDUCTION OF DENOMINATE NUMBERS

Change :

1. $5\frac{1}{3}$ yd. to feet.
2. 90 in. to feet.
3. 3 yd. 2 ft. to feet.
4. .5 rd. to inches.
5. 25 ft. to yards.
6. 5.5 hr. to minutes.
7. .5 mi. to rods.
8. 3.5 gal. to pints.
9. $\frac{1}{3}$ day to minutes.
10. .25 bu. to quarts.
11. $\frac{3}{4}$ pk. to quarts.
12. 2 lb. 8 oz. to ounces.
13. $\frac{3}{4}$ cwt. to pounds.
14. .5 yd. 1 ft. to inches.
15. .75 mi. to rods.
16. .25 bu. to pints.
17. 3.5 pk. to quarts.
18. 2 yd. 1.5 ft. to inches.
19. 3.5 min. to seconds.
20. 48 qt. to pecks.
21. 64 pt. to bushels.
22. 64 oz. to pounds.

Written Work

1. Change 3 gal. 3 qt. 1 pt. to pints.

gal. qt. pt.
3 3 1
4
12
+3
15, number of quarts.
2
30
+1
31, number of pints.

Observe that 4 qt. is really the multiplicand and 3 the multiplier in finding the first product; and that 2 pt. is really the multiplicand and 15 the multiplier in finding the second product. In considering the numbers abstractly, however, either factor may be regarded as the multiplicand and the arrangement as indicated saves time.

2. Change .875 gallon to pints.

.875
4
3.500, number of qt.
2
1.00, number of pt.

Since there are 4 qt. in a gallon, in .875 of a gallon there are .875 of 4 qt., or 3.5 qt. Since there are 2 pt. in 1 qt., in .5 of a quart there is 1 pt. The answer is 3 qt. 1 pt.

Change:

3. 15 lb. 8 oz. to ounces.
4. 96 ft. 5 in. to inches.
5. 5.5 bu. to quarts.
6. 3.5 pk. to pints.
7. 18 cwt. 25 lb. to pounds.
8. 23 hr. 16 min. to minutes.
9. 8.3 mi. to yards.
10. .75 yd. to inches.
11. $4\frac{4}{5}$ T. to pounds.
12. $7\frac{3}{4}$ min. to seconds.
13. 6.5 L. T. to pounds.
14. 63.5 gal. to pints.
15. $\frac{7}{8}$ bu. to quarts.
16. $10\frac{3}{4}$ bu. to pecks.

17. Change 266 quarts to bushels.

8)266
4)33 no. of pk. + 2 qt.
8 no. of bu. + 1 pk.
8 bu. 1 pk. 2 qt.

There are $\frac{1}{8}$ as many pecks as quarts, that is, 33 pk. + 2 qt. There are as many bushels as pecks, that is, 8 bu. + 1 pk. Hence, 266 qt. = 8 bu. 1 pk. 2 qt.

Change to higher denominations:

18. 342 inches.
19. 6625 yards.
20. 5281 feet.
21. 2043 seconds.
22. 1033 ounces Av.
23. 347 cwt.
24. 6095 pounds.
25. 16857 rods.
26. 11097 qt. (Dry).
27. 952 pt. (Liquid).
28. 43920 in.
29. 6875 sec.
30. 56.5 pk.
31. 684.5 rd.
32. $964\frac{3}{5}$ min.

33. How many gallons of milk will a family consume in 75 days, if they use 2 qt. 1 pt. daily?

34. How much is received for $1\frac{1}{2}$ bushels of chestnuts at 8 cents a quart?

35. How much will 15 turkeys, averaging $14\frac{3}{4}$ lb. each, cost at 18 cents a pound?

36. If 100 tons of coal are bought by the long ton, at \$2.24 a ton, and sold by the short ton at the same price, how much is gained?

37. At 20 cents an hour, how much will a man earn in 26 days, working each day from 8 A.M. to 5 P.M., allowing 1 hour for lunch?

38. If a flour mill grinds wheat at the rate of 1 pint in 5 seconds, in how many hours and minutes will it grind 21,600 bushels?

39. A train goes 104 miles in 3 hours and 15 minutes. What is the rate per hour?

40. At 6 cents a foot find the length in miles and rods of a telephone wire that costs \$4672.80.

41. If a man's step averages 2 ft. 6 in., how far will he travel in taking 6600 steps?

Relations of denominate measures.

1. $\frac{3}{4}$ pk. is what decimal part of a bushel?

$\frac{3}{4}$ pk. $= 6$ qt.

6 qt. $= \frac{6}{32}$ bu. $= .1875$ bu.

2. 3 ft. 2 in. is what fractional part of a rod?

3 ft. 2 in. $= 38$ in.

1 rod $= 198$ in.

3 ft. 2 in. $= \frac{38}{198}$, or $\frac{19}{99}$ rd.

Find the fractional part:

3. $2\frac{2}{3}$ hr. is of 1 day.

4. $7\frac{1}{2}$ ft. is of 1 rod.

5. $3\frac{1}{2}$ qt. is of 1 gallon.

6. $1\frac{1}{2}$ pt. is of 3 qt.

7. $2\frac{1}{2}$ in. is of 10 ft.

8. $1\frac{1}{2}$ qt. is of 2 gal.

Find the fractional part:

9. 15 hundredweight is of 1 ton.

10. 3.5 quarts is of 1 bushel.

11. 280 rods is of 1 mile.

12. 37 pounds 8 ounces is of 1 hundredweight.

13. 440 yards is of 1 mile.

Find the decimal part:

14. 16 hours 48 minutes is of 1 day.

15. 1 foot 8 inches is of 1 rod.

16. 183 pounds 12 ounces is of 1 ton.

17. 15 minutes 50 seconds is of 1 hour.

18. A machinist works 10 hr. per day in summer and $8\frac{1}{3}$ hr. per day in winter. If his wages in summer are $3.35 per day, at the same rate find his wages per day in winter.

ADDITION AND SUBTRACTION

1. Find the sum of 2 gal. 3 qt. 1 pt., 4 gal. 1 qt. 1 pt., 7 gal. 1 pt., 5 gal. 3 qt.

gal.	qt.	pt.
2	3	1
4	1	1
7	0	1
5	3	0
20	0	1

The sum of the pints = 3 pt. = 1 qt. and 1 pt.
The sum of the quarts + 1 qt. carried = 8 qt. = 2 gal. 0 qt.
The sum of the gallons + 2 gal. carried = 20 gal.

Add:

2. 14 bu. 2 pk., 5 bu. 6 qt., 7 qt. 1 pt., 9 bu. 6 qt.

3. 5 T. 11 cwt., 4 T. 15 cwt. 60 lb., 11 T. 80 lb., 19 T. 3 cwt. 64 lb.

4. 9 yr. 120 da. 8 hr., 12 yr. 104 da. 17 hr., 14 da.

5. 3 wk. 6 da. 15 hr., 4 wk. 3 da. 9 hr., 7 wk. 5 da. 14 hr.

6. From 7 bu. 3 pk. 3 qt. take 3 bu. 1 pk. 5 qt.

bu.	pk.	qt.
7	3	3
3	1	5
4	1	6

1 pk. or 8 qt. + 3 qt. = 11 qt.; 11 qt. − 5 qt. = 6 qt.; 2 pk. − 1 pk. = 1 pk.; 7 bu. − 3 bu. = 4 bu.

Subtract:

	mi.	rd.	yd.	ft.
7.	80	120	0	12
	57	245	0	14

	gal.	qt.	pt.
8.	23	0	1
	9	3	0

9. From 18 hr. take 9 hr. 16 min. 45 sec.

Finding the **difference in time** between two dates is the most practical application of subtraction of denominate numbers.

10. Find the difference in time between November 15, 1903, and August 12, 1905.

yr.	mo.	da.
1905	8	12
1903	11	15
1	8	27

Aug. 12, 1905, is represented as the 12th day of the 8th month of 1905, and Nov. 15, 1903, as the 15th day of the 11th month of 1903.

1 mo. or 30 da. + 12 da. = 42 da.; 42 da. − 15 da. = 27 da.; 1 yr. or 12 mo. + 7 mo. = 19 mo.; 19 mo. − 11 mo. = 8 mo.; 1904 yr. − 1903 yr. = 1 yr.

Subtract:

	yr.	mo.	da.
11.	1908	7	12
	1901	9	15

	yr.	mo.	da.
12.	1905	9	1
	1890	8	15

13. How many years, months, and days old is each pupil in the room?

14. General Robert E. Lee was born January 19, 1807, and General Ulysses S. Grant April 27, 1822. How old was each at the close of the Civil War, April 9, 1865? How much older was General Lee than General Grant?

15. How old is a man to-day who was born July 3, 1882?

MULTIPLICATION AND DIVISION

1. Multiply 3 wk. 5 da. 9 hr. by 7.

wk.	da.	hr.
3	5	9
		7
26	2	15

7×9 hr. $= 63$ hr. $= 2$ da. and 15 hr.; 7×5 da. $= 35$ da.; 35 da. $+ 2$ da. $= 37$ da. $= 5$ wk. and 2 da.; 7×3 wk. $= 21$ wk.; 21 wk. $+ 5$ wk. $= 26$ wk. Hence the answer is 26 wk. 2 da. 15 hr.

Multiply:

2. 3 gal. 2 qt. 1 pt. by 3.
3. 12 bu. 3 pk. 3 qt. by 6.
4. 15 T. 5 cwt. 12 oz. by 10.
5. 27 wk. 3 da. 14 hr. by 9.
6. 23 mi. 124 rd. 11 ft. 4 in. by 12.
7. Divide 54 T. 15 cwt. 72 lb. by 12.

T.	cwt.	lb.
12)54	15	72
4	11	31

54 T. $\div 12 = 4$ T. and 6 T. remaining; 6 T. $= 120$ cwt.; 120 cwt. $+$ 15 cwt. $= 135$ cwt.; 135 cwt. $\div 12 = 11$ cwt. and 3 cwt. remaining; 3 cwt. $= 300$ lb.; 300 lb. $+ 72$ lb. $= 372$ lb.; 372 lb. $\div 12 = 31$ lb.

Divide:

8. 18 wk. 5 da. 21 hr. by 5.
9. 188 gal. 1 pt. by 7.
10. 88 bu. 3 pk. 4 qt. by 9.
11. 61 yr. 11 mo. 18 da. by 11.
12. 86 T. 3 cwt. 44 lb. by 6.
13. Find the cost of 19 gross of pencils at 10¢ a dozen.
14. What is the contents in cubic feet of a cistern that is 18 ft. 6 in. long, 11 ft. 5 in. wide, and 8 ft. 3 in. deep?
15. How many packages, weighing 5 ounces each, can be made from 5 pounds of candy?

REVIEW

1. If a watch gains 18 seconds in a day, how much too fast will it be in three weeks?

2. How many barrels, each holding 2 bushels and 3 pecks, will be required to pack 88 bushels of apples?

3. How many bushels of potatoes are necessary to plant $8\frac{7}{8}$ acres, allowing 6 bu. 1 pk. to the acre?

4. A merchant sells linseed oil at 12¢ a pint that cost him 56¢ a gallon. Find his profits on 45 gallons 3 quarts.

5. 5 car loads of coal weigh: 57,698 lb., 49,875 lb., 63,545 lb., 49,897 lb., and 54,273 lb. Find the number of tons, hundredweight, and pounds in all.

6. 4 men buy a plot of land that has 222 feet 8 inches street frontage. Allowing for an alley 20 feet in width in the center, what is the width of each man's lot if they divide the plot equally?

7. A force pump in a coal mine lifts $76\frac{5}{8}$ gallons of water to the surface per minute. Find the weight of the water pumped out in one day.

8. If 3 pounds 4 ounces of coal are consumed in generating power to lift 5 gallons of water in problem 7, find the number of tons of coal consumed each day.

9. A Kentucky farmer clipped $542\frac{1}{2}$ pounds of mohair from 70 Angora goats. Find the average clip from each goat and its value at $\$.67\frac{1}{2}$ per pound.

10. An automobile runs $2\frac{7}{8}$ miles in 5 minutes. At that rate, find the distance in miles, rods, and feet it runs in 1 hour 35 minutes.

11. A pencil factory makes $6\frac{3}{4}$ gross of pencils per hour. Find the number of dozen made in 26 days of $9\frac{1}{2}$ hours each.

PRACTICAL MEASUREMENTS

MEASURES OF LENGTH

1. Measure the length of your desk; the length of the room; the length of the blackboard; the height of the window from the floor.

2. In what are these short lengths measured?

To the Teacher. — Secure a tape measure 50 feet long.

3. Measure the distance around the schoolroom in feet and fractions of a foot. How many yards is it around the room?

4. Measure the distance around the school grounds in rods, feet, and inches.

5. Take $16\frac{1}{2}$ ft. of the tape measure and measure 10 rods along the public road or street.

6. $320 \times 16\frac{1}{2}$ ft. = how many feet?

   1760×3 ft. = how many feet?

7. How many feet equal a mile? how many yards?

8. James walks $1\frac{3}{4}$ miles to school each day. How many rods does he walk? how many feet?

9. How many rods equal 5280 ft.? $\frac{3}{4}$ of a mile? 3560 ft.?

10. Mary walks $\frac{3}{4}$ of a mile to school each day. How many miles does she walk in going to and from school in 180 days?

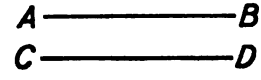
11. Henry walks .8 of the number of miles Mary walks. Find the distance Henry walks in a term if he attends 160 days.

12. Write the table of length.

13. A train averages 35.3 miles per hour. How many miles, rods, etc., does the train run in 6 hours and 45 minutes?

14. What part of a mile is a foot? a yard? a rod?

15. Examine lines *AB* and *CD*.

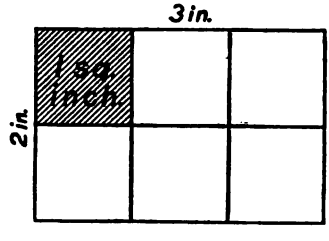


Observe that they are straight lines and cannot meet, however far they may be extended. Such lines are called **parallel lines.**

MEASURES OF SURFACE

1. Name six different surfaces in the schoolroom.

2. How many dimensions has every surface? How does a surface differ from a line?



3. Draw on a scale of $\frac{1}{2}$, a rectangle 3 inches long and 2 inches wide. Divide it by lines into square inches. How many square inches are there in the first row? in the second? in the rectangle? What is the *unit* of measure in this surface?

Observe that $2 \times 3 \times 1$ *sq. in.* $= 6$ sq. in.

The **area of a rectangle** *is found by multiplying its unit of measure by the product of its two dimensions.*

4. Draw a rectangle 2 ft. by 6 ft. Divide it into square feet.

5. Draw a square a foot on a side. Mark off the sides into 12 equal parts and connect them by straight lines. How many square inches equal a square foot?

6. Draw on the blackboard a line 4 feet long. From each end draw lines in the same direction 3 feet in length, making square corners with the 4-foot line. Connect by a straight line the ends of the 3-foot lines.

7. Are the sides of the figure straight? Are the corners equal in size? Find the area of the figure.

8. What is a right angle? a rectangle? (p. 112).

9. Show by a diagram the number of square feet in a square yard.

10. Draw a diagram on a scale of 1 inch to 3 feet to represent a rectangle 24 ft. long and 18 ft. wide. Find its area.

Draw diagrams on scales suitable to the size of your tablet or slate and find the surface of each of the following:

11. A rectangle 20 ft. by 24 ft.

12. A flower bed 16 ft. by 8 ft.

13. A floor 16 ft. long and 14 ft. wide.

14. A wall 15 yd. long and 5 yd. high.

15. By actual measurement find the number of square feet in the floor, the door, the blackboard, and the walls of the schoolroom.

16. In what denominations did we find the lengths and widths of the problems just given?

Land is measured in *acres*, *square rods*, *square feet*, etc.

17. Measure a square yard on your playground. How long is it? how wide?

18. Measure the length and width of your school grounds in rods and feet.

19. Since $16\frac{1}{2}$ feet equal 1 rod, how many yards equal 1 rod? How many square yards equal 1 square rod?

20. Since $16\frac{1}{2}$ feet equal 1 rod, how many square feet equal 1 square rod?

21. A field is 70 rods long and 40 rods wide. How many square rods are there in it? how many acres?

22. Memorize this table:

144 square inches (sq. in.)	**= 1 square foot (sq. ft.)**
9 square feet	**= 1 square yard (sq. yd.)**
$30\frac{1}{4}$ square yards	**= 1 square rod (sq. rd.)**
160 square rods	**= 1 acre (A.)**
640 acres	**= 1 square mile (sq. mi.)**

1 A. = 160 sq. rd. = 4840 sq. yd. = 43,560 sq. ft.

Change:

23. 2700 sq. yd. to sq. ft.

24. 50 sq. ft. to sq. in.

25. 1600 sq. rd. to A.

26. $1\frac{5}{8}$ A. to sq. rd.

27. 800 sq. yd. to sq. rd.

28. $5\frac{3}{4}$ A. to sq. ft.

29. A farm is 90 rods long and 60 rods wide. Find the number of acres in it. Find its cost at \$60 per acre.

30. A lot 100 ft. square has a house 36 ft. by 42 ft. located on it. The remaining space is lawn. Find the number of square feet of lawn. Draw diagram.

31. A concrete sidewalk in front of the lot is 4 ft. wide. Find its cost at 19¢ per square foot.

32. Find the cost of a flagstone walk, 135 ft. long and 6 ft. wide, at 21¢ per square foot.

33. City lots are sometimes sold by the square foot. Find the cost of a lot in Pittsburg 21 ft. by 70 ft. at \$27.50 per square foot.

34. A farm 160 rods long and 120 rods wide is sold in two pieces, $\frac{5}{8}$ of it at \$60 per acre, and the remainder at \$50 per acre. Find the amount of the entire sale.

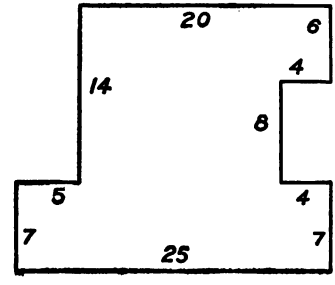
35. An Iowa farmer owns a farm a mile square. How many acres has he? Find its value at \$85 per acre.

36. A western wheat field 100 rods long and 80 rods wide yields 880 bushels of wheat. Find the average yield per acre.

37. City lots are usually sold by the front foot. Find the cost, at \$20 per foot front, of a lot 25 ft. front by 120 ft. deep. Find the cost per square foot.

38. A four-room school building has a slate blackboard 24 ft. by 4 ft. in each room. Find the total cost of the blackboard at 23¢ per square foot.

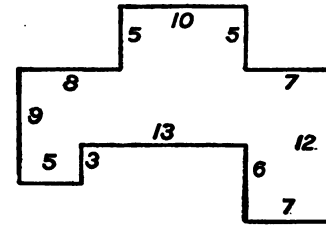
39. The area of a field in the form of a rectangle is 8 acres. If one side is 32 rods, what is the other?



These diagrams represent pieces of land. The dimensions are given in rods, and the angles are all square.

40. Divide the first piece into 3 rectangles and find (1) how many square rods there are in each; (2) the perimeter of each; (3) the area of the entire piece in acres.

41. Divide the second piece into rectangular lots, and find (1) the perimeter of each; (2) the area of each; (3) the area of the entire piece.



PAINTING AND PLASTERING

Painting, plastering, and kalsomining are generally measured by the **square yard.** In some localities an allowance is made for doors and windows, but there is no uniform rule in practice.

1. How much will it cost to paint a ceiling 18 ft. long and 15 ft. wide at 10¢ per square yard?

2. How much will it cost to kalsomine a hall 30 ft. long, 9 ft. wide, and 15 ft. high, at 5¢ per square yard? (Observe that the perimeter of the hall is 78 ft.)

3. How many square yards of plastering are there in a room 21 ft. long, 18 ft. wide, and 12 ft. high, making no allowance for openings?

4. How much will it cost, at 15¢ a square yard, to plaster a room 24 ft. $\times$ 19½ ft. $\times$ 15 ft.?

5. A public hall is 120 ft. $\times$ 66 ft. $\times$ 22½ ft. How much will it cost to paint the walls and ceiling at 10¢ per square yard?

THE RIGHT TRIANGLE

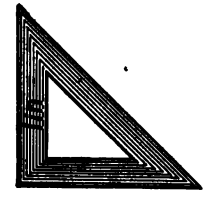
1. Draw on the blackboard a rectangle 12 inches long and 8 inches wide. Connect the opposite corners by a straight line.

This line is called the **diagonal** of the rectangle.

2. Into how many parts have we divided the rectangle? Shade one of the parts with chalk. How many angles are there in each part? how many right angles?

A **triangle** is a surface having three sides and three angles.

A **right triangle** is a triangle having one right angle.

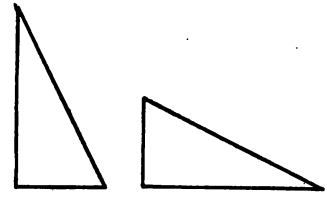


The **base** of a triangle is the side on which it is assumed to stand.

The **altitude** of a triangle is the line that meets the base line at a right angle.

To THE TEACHER. — As an aid in drawing have each pupil, if possible, get a right triangle as here shown.

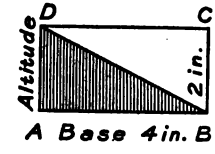
3. Point out the base and altitude in the triangles at the right.



4. Fold a rectangular piece of paper, as $ABCD$, on its diagonal. Observe:

(1) That the rectangle $ABCD$ and the triangle ABD have the same base and altitude.

(2) That the area of the triangle is just $\frac{1}{2}$ the area of the rectangle.



Hence, $\frac{1}{2}$ of $4 \times 2 \times 1$ *sq. in.* = 4 sq. in.; area.

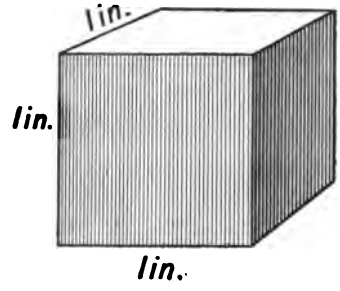
The **area of a right triangle** *equals the unit of measure multiplied by* $\frac{1}{2}$ *the product of the base and altitude.*

Draw on a scale suitable to your paper and find the area of the following right triangles in square inches:

5. Base 10 in., altitude 8 in.
6. Base 12 in., altitude 6 in.
7. Base 25 in., altitude 18 in.
8. Base 36 in., altitude 24 in.

9. Find the area of a field in the form of a right triangle whose base is 80 rods and altitude 40 rods.

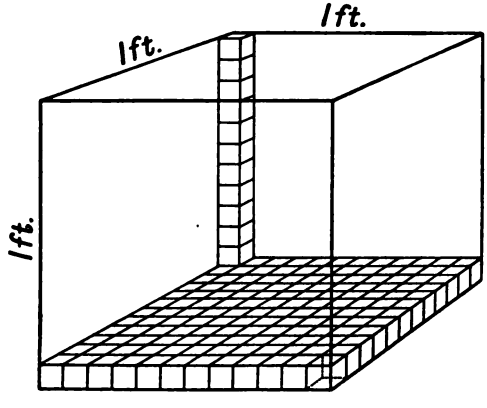
MEASURES OF VOLUME



1. How many dimensions has the cube? Name them.

2. What dimensions has a line? What dimensions has a surface? a solid?

3. Name the different units of measure in which the length of a line may be expressed.



4. Name the different units of square measure in which surface may be expressed.

5. What cubic unit have we in the first cube?

6. If a cube is 1 foot on an edge, what is the cubic unit?

7. Draw a square 1 foot on a side. Show that it contains 144 square inches.

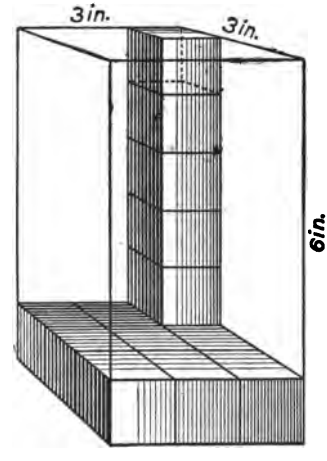
8. Observe that 144 cubes 1 inch on an edge can be placed on a surface 1 foot square. How many layers of such cubes will it take to make a cube 1 foot on an edge?

9. How many cubic inches equal 1 cubic foot?

10. How many surfaces has a cube?

11. Show that all the surfaces of an inch cube are the same in area; of a 2-inch cube; of a 9-inch cube.

12. Examine carefully the figure. Observe:



(1) That the surface of the face upon which it rests contains 9 square inches.

(2) That the first layer of units of volume contains 9 cubic inches.

(3) That the whole solid, if 6 inches high, contains 6×9 cubic inches, or 54 cubic inches.

13. How many 1-inch cubes are there in the first layer? how many in the solid?

14. What is the shape of the surfaces of the solid? Is each surface a rectangle?

A **rectangular solid** is a solid whose surfaces are all rectangles.

15. Observe that the number of inch cubes in the solid is equal to the product of its three dimensions.

16. What is the unit of measure in the solid?

Observe that $3 \times 3 \times 6 \times 1$ *cu. in.* = 54 cu. in.

The **contents of a rectangular solid** *equals the unit of measure multiplied by the product of its three dimensions.*

To THE TEACHER. — Secure 144 1-inch cubes.

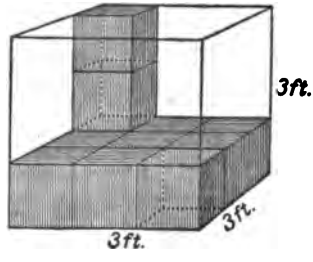
17. Build a cube 2 inches on an edge.

18. Build a cube 4 inches on an edge.

19. Compare the 4-inch cube with the 2-inch cube.

20. Give the different units of measure of surface; of length; of contents.

21. Find the contents of a box 3 ft. long, 2 ft. wide, and $1\frac{1}{2}$ ft. high.



22. Observe the cube. What is its length? width? height?

23. How many 1-foot cubes does it contain?

A cube 3 ft. on an edge is called a **cubic yard.**

24. Memorize this table:

1728 cubic inches (cu. in.)	**= 1 cubic foot (cu. ft.)**
27 cubic feet	**= 1 cubic yard (cu. yd.)**

A **cart load** of earth is considered 1 **cubic yard.**

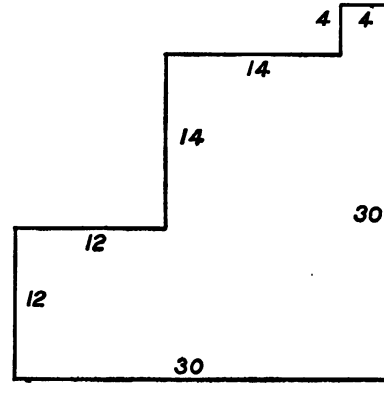
PRACTICAL APPLICATIONS

Excavations are estimated by the **cubic yard.**

1. Find the cost, at 30¢ per cubic yard, of excavating a cellar 36 ft. in length, 24 ft. in width, and 4 ft. in depth.

$36 \times 24 \times 4 \times 1$ *cu. ft.* = 3456 cu. ft., the contents of the cellar.
3456 cu. ft. ÷ 27 = 128, number of cu. yd.
128 × \$.30 = \$38.40, cost of excavation.

This diagram shows the outline of a cellar 5 ft. deep. Its dimensions are given in feet.



2. Find its area in square feet.

3. Find the length of its walls.

4. What is the cost of excavating it at 32¢ per cubic yard?

5. How much will it cost to cement the floor at $.90 per square yard?

6. If a boy inhales 24 cubic inches of air at a breath, how many times must he breathe to inhale 1 cubic foot?

7. 29 pupils and their teacher occupy a schoolroom 30 ft. in length, 24 ft. in width, and 12 ft. in height. What is the average number of cubic feet of air for each person?

8. A city lot of $37\frac{1}{2}$ ft. by 120 ft. is to have a layer of earth 1 ft. thick over its surface. Find the number of loads needed and its cost at 25¢ per load.

9. A dining room is 13 ft. by 18 ft. and has a rug on it 9 ft. by 15 ft. Find the surface not covered by the rug.

10. If the rainfall on a certain day was $2\frac{1}{4}$ inches, find the number of cubic inches that fell on a lot 25 feet wide and 100 feet long. Find the number of gallons.

11. Find the cost of digging a ditch, 60 rods long, $3\frac{1}{2}$ feet wide, and 6 feet deep, at 60¢ per cubic yard.

12. Memorize:

A gallon = 231 cu. in.
A bushel = 2150.42 cu. in.
1 bushel = $1\frac{1}{4}$ cu. ft. (nearly)

13. Compare a 3-inch cube with a 4-inch cube. If a 2-inch cube weighs 6 ounces, how much will a 4-inch cube of the same material weigh?

14. A bin is 8 ft. long, 6 ft. wide, and 4 ft. deep. Estimate quickly about the number of bushels of wheat or oats it will hold.

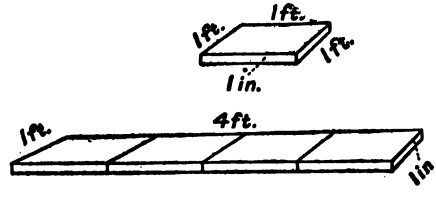
15. A farmer has a tank 12 ft. long, 8 ft. wide, and 6 ft. deep. How many gallons of water will it hold?

16. How much larger is a farm 80 rods square than a farm 60 rods square? Draw diagrams on a suitable scale to represent this.

17. The base of a rectangular tank is 48 sq. ft. and the volume is 192 cu. ft. Find the height.

18. What is the area of each surface of a cube 8 ft. on an edge? the entire surface?

MEASUREMENT OF LUMBER



A board 1 foot square and 1 inch thick or less is a **board foot.**

A *board foot* is the *unit* in measuring lumber.

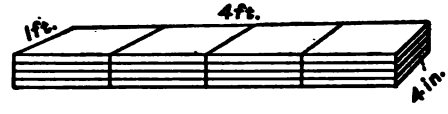
1. Draw on the blackboard a figure to represent a board 4 feet long, 1 foot wide, and 1 inch thick.

2. Show that this board contains 4 board feet.

3. How many board feet are there in a sill 4 ft. long, 1 ft. wide, and 4 in. thick?

Observe:

(1) That the sill is equal to 4 boards 4 ft. long, 1 ft. wide, and 1 in. thick.



(2) That each board contains 4 board feet.

$4 \times 4 \times 1$ *board foot* = 16 board feet.

The **number of board feet in a piece of lumber** ***equals the number of square feet in one surface multiplied by the thickness in inches.***

Find the number of board feet in the following:

4. A plank 12 ft. long, 12 in. wide, and 2 in. thick.
5. A board 12 ft. long, 9 in. wide, and 1 in. thick.
6. A plank 15 ft. long, 12 in. wide, and 3 in. thick.
7. A plank 16 ft. long, 18 in. wide, and 2 in. thick.
8. A sill 20 ft. long, 10 in. wide, and 6 in. thick.
9. A sill 30 ft. long and 12 in. square.

Buying and selling lumber.

Lumber is usually sold at so much per 1000 (M.) *board feet.*

Find the cost of:

1. 5000 ft. poplar at $40 per M.
2. 500 ft. hemlock at $24 per M.
3. 10,850 ft. Georgia pine at $24 per M.
4. 8000 ft. white pine at $50 per M.

Small bills of lumber are usually estimated at so much per *board foot.*

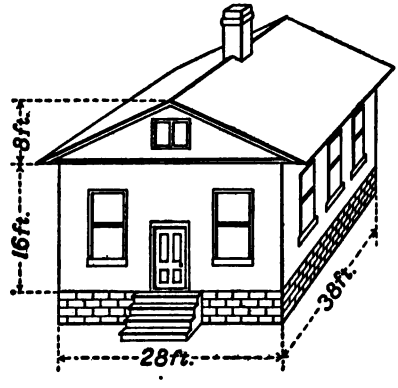
5. Show that lumber at $40 per M. = $.04 per board foot; at $27 per M. = $.027 per board foot.

6. Make out a receipted bill to Henry James for the following: 365 ft. hemlock at $25 per M., 780 ft. white pine at $40 per M., 980 ft. yellow pine at $29 per M.

The dimensions 10 ft. by 6 in. by 10 in. are commonly written $10' \times 6'' \times 10''$.

Estimate the cost of the following at $28 per M.:

7. 4 sills $4'' \times 8'' \times 24'$
8. 6 girders $6'' \times 10'' \times 16'$
9. 2 posts $6'' \times 9'' \times 10'$
10. 8 beams $3'' \times 8'' \times 20'$
11. 60 joists $3'' \times 8'' \times 20'$
12. 90 studding $2'' \times 6'' \times 16'$
13. 90 planks $2'' \times 10'' \times 14'$
14. 60 rafters $2'' \times 4'' \times 24'$



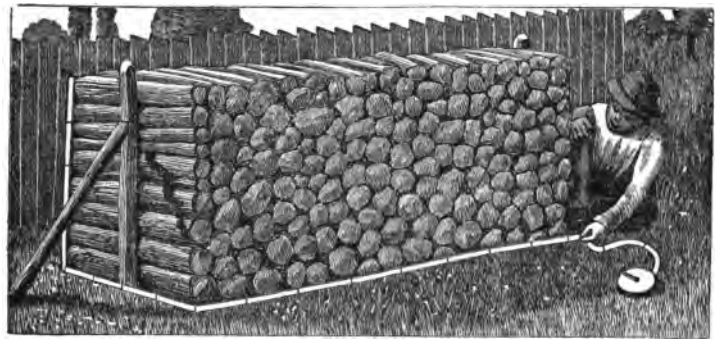
15. Observe the dimensions of the school building. What is the height of the sides of the building?

16. Find the number of board feet of siding needed for the sides and the two ends of the same height as the sides, making no allowance for openings.

17. The triangular parts at the top of the house, in front and in back, are called *gables*. Each gable can be divided by a line through the center of its base into two right triangles. How many board feet of siding are necessary for the two gables?

18. Find the cost of painting the siding at 10 cents per square yard.

MEASURING WOOD



A pile of wood, of 4-foot sticks, 8 ft. in length and 4 ft. in height, is called a **cord of wood.**

$4 \times 4 \times 8 \times 1$ *cu. ft.* = 128 cu. ft. = 1 cord of wood.

1. How many cords are there in a pile of 4-foot wood, 160 feet long and 4 feet high?

2. Two men cut several piles of 4-foot wood that measure in all 640 feet in length and 4 feet in height. How many cords do they cut and how much do they receive for the work at $5.50 per cord?

Wood is frequently cut for house purposes into short lengths from 16 inches to 2 feet. The price of such a cord varies according to the length of the sticks.

A **cord of short wood** *is measured by multiplying the length in feet by the height in feet and dividing the product by* 32.

3. At a school building there is a pile of 16-inch wood 80 ft. long and 4 ft. high. Find its cost at $1.50 per cord.

4. Two men cut 4 cords of 2-foot wood each day for 16 days. Find the cost of the cutting at 70 cents per cord.

5. One side of a pile of 2-foot wood contains 400 square feet. Find the number of cords it contains.

REVIEW OF PRACTICAL MEASUREMENTS

1. How many tiles 12 in. square will be required to lay a floor 36 ft. by 15 ft.?

2. What is the length of a board walk that is 4 ft. 8 in. wide and contains 1350 sq. ft.?

3. How many cubic yards of earth must be removed in digging a cellar 36 ft. long, 26 ft. wide, and 8 ft. deep?

4. Find the cost of covering the floor of a hall 45 ft. long and 30 ft. wide with matting, a yard wide, at 70 cents a yard.

5. How many times will the wheel of an engine 9 ft. in circumference turn in going 3000 miles?

6. Find the cost of 30 boards 16 ft. long, 12 in. wide, and 1 in. thick, at 5¢ a board foot.

7. At $.80 a bushel what is the value of a bin of wheat 16 ft. long, 8 ft. wide, and 4 ft. deep?

8. What is the number of gallons in a tank 12 ft. long, 10 ft. wide, and 8 ft. deep?

9. How much will it cost to cement the floor of a cellar 50 ft. long and 28 ft. wide, at $1.08 a square yard?

10. At 7¢ a square yard, how much will it cost to paint the four sides of a building 50 ft. long, 20 ft. wide, and 15 ft. high?

11. My farm is in the form of a rectangle, and contains 40 acres. What is its width, if its length is 128 rods?

12. What will be the cost of plastering the ceiling of a room 22 ft. by 18 ft. at 11¢ a square yard?

13. A rectangular field contains 5 acres. If its length is 80 rods, what is its width?

14. How many cakes of soap 4 in. by 3 in. by 2 in. can be packed in a box whose inside dimensions are 2 ft., 3 ft., and 4 ft.?

15. Find the cost of digging a cellar 42 ft. long, 30 ft. wide, and 6 ft. 3 in. deep, at 40 cents a cubic yard.

16. How much flooring 1 inch thick will be required to lay the first floor of a house 22 ft. by 36 ft., no allowance being made for waste, and how much will it cost at $30 per M.?

17. The length of a field is 80 rods, and its width is 30 rods. How many acres are there in the field?

18. What is the number of bushels in a bin 20 ft. long, 16 ft. wide, and 8 ft. deep?

19. A tank 9 ft. square and 8 ft. deep contains how many gallons?

20. A building lot 100 foot front contains 15,000 sq. ft. What is its depth?

21. A baseball ground 160 yd. by 170 yd. has a tight board fence around it 8 ft. high. How much will the painting of the outside of the fence cost at $5\frac{1}{2}$ cents a square yard?

22. The area of a right triangle is 560 sq. ft., and its altitude is 28 ft. What is the base of the triangle?

23. How much will it cost to excavate a street 800 ft. long and 50 ft. wide, to a depth of 18 in., at 36 cents a load?

24. A plot of ground in the form of a square is 100 ft. on each side. A straight walk 8 ft. wide divides it into 2 equal parts — a lawn for flowers and a garden for vegetables. In the lawn there is a flower bed 5 ft. by 8 ft. Draw the plot.

25. Find the perimeter of the plot; of the lawn; of the garden; of the flower bed; of the walk.

Find the area in square yards:

26. Of the plot.

27. Of the lawn.

28. Of the flower bed.

29. Of the walk.

30. How much will it cost to fence the plot at $3\frac{3}{4}$ per rod?

31. How much will it cost to pave the walk at $1.55 per square yard?

32. How much will it cost to spade the flower bed at 5 cents per square yard?

33. How much will it cost to sod the lawn, excluding the flower bed, at $0.25 per square yard?

34. A board 16 ft. long contains 9 sq. ft. Find its width.

35. A room is 20 ft. long, 16 ft. wide, and 10 ft. high. How much will it cost to plaster the walls and ceiling at 20 cents a square yard?

36. How many gallons of water are there in a tank 12 ft. long, 8 ft. wide, and 6 ft. deep if it is half full?

37. Find the cost of 40 boards, each 14 ft. long, 18 in. wide, and 1 in. thick, at $20 per M.

38. A city 5 miles long and 3 miles wide is equal in area to how many farms of 160 acres each?

39. How many sods 16 in. square will be required to turf a lawn 106 ft. 8 in. long and 50 ft. wide?

40. What will be the cost of painting the outside of a house 48 ft. long, 30 ft. wide, and 20 ft. high, with 2 coats of paint, at 18 cents a square yard?

PERCENTAGE

Per cent means **by the hundred** or **hundredths.** The sign for it is *%*.

We may express the per cent of a number either as a *common fraction* or a *decimal.*

Thus, $6\% = \frac{6}{100} = .06$; 6% of 500 means $\frac{6}{100}$ of 500, which equals 30; or, expressed decimally, .06 of 500 = 30.

2% of a number means $\frac{2}{100}$ or .02 of a number.

25% of a number means $\frac{25}{100}$ or .25 of a number.

1. What term in common fractions corresponds to the number before the sign *%*? to the sign *%*?

2. What expresses the *numerator* and what indicates the *denominator* of the fractions represented by the following:

1%?	20%?	40%?	90%?
6%?	30%?	75%?	100%?

3. Find 6% of 100.

$\frac{6}{100}$ of 100 = 6; or .06 × 100 = 6.

Find:

4. 5% of 100	10. 10% of 100	16. 8% of 75
5. .05 of 100	11. 25% of 100	17. .08 of 75
6. $\frac{5}{100}$ of 100	12. .25 of 400	18. $\frac{8}{100}$ of 75
7. 6% of 150	13. 3% of 60	19. $33\frac{1}{3}$ of 300
8. .06 of 150	14. .03 of 60	20. $\frac{33\frac{1}{3}}{100}$ of 300
9. .10 of 100	15. $\frac{8}{100}$ of 60	21. $33\frac{1}{3}\%$ of 300

Changing per cents to equivalents.

Since $5\% = .05 = \frac{5}{100} = \frac{1}{20}$, these expressions may be called **equivalents.**

1. Give the fractional and decimal equivalents for 10 %; 6 %; 4 %; 20 %; 25 %.

Read the following equivalents:

2. $\frac{20}{100}$, 20%, .20, $\frac{1}{5}$

3. $\frac{12\frac{1}{2}}{100}$, $12\frac{1}{2}\%$, $.12\frac{1}{2}$, $\frac{1}{8}$

4. $\frac{40}{100}$, 40%, .40, $\frac{2}{5}$

5. $\frac{37\frac{1}{2}}{100}$, $37\frac{1}{2}\%$, $.37\frac{1}{2}$, $\frac{3}{8}$

6. $\frac{80}{100}$, 80%, .80, $\frac{4}{5}$

7. $\frac{87\frac{1}{2}}{100}$, $87\frac{1}{2}\%$, $.87\frac{1}{2}$, $\frac{7}{8}$

8. Change the fractions $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$ to their equivalent decimals and per cents; also $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$.

$\frac{1}{5} = 5\overline{)1.00}$ or 20 %
$\frac{2}{5} = 2 \times .20 = .40$ or 40 %
$\frac{3}{5} = 3 \times .20 = .60$ or 60 %
$\frac{4}{5} = 4 \times .20 = .80$ or 80 %

$\frac{1}{8} = 8\overline{)1.00}$ or $12\frac{1}{2}\%$
$\frac{3}{8} = 3 \times .12\frac{1}{2} = .37\frac{1}{2}$ or $37\frac{1}{2}\%$
$\frac{5}{8} = 5 \times .12\frac{1}{2} = .62\frac{1}{2}$ or $62\frac{1}{2}\%$
$\frac{7}{8} = 7 \times .12\frac{1}{2} = .87\frac{1}{2}$ or $87\frac{1}{2}\%$

Change to their equivalent decimals and per cents:

9. $\frac{1}{2}$
10. $\frac{1}{3}$
11. $\frac{2}{3}$
12. $\frac{1}{4}$
13. $\frac{3}{4}$
14. $\frac{1}{6}$
15. $\frac{5}{6}$
16. $\frac{1}{10}$
17. $\frac{3}{10}$
18. $\frac{7}{10}$
19. $\frac{9}{10}$
20. $\frac{1}{5}$
21. $\frac{2}{5}$
22. $\frac{3}{5}$
23. $\frac{4}{5}$
24. $\frac{1}{8}$
25. $\frac{3}{8}$
26. $\frac{5}{8}$
27. $\frac{7}{8}$
28. $\frac{1}{16}$

Give the products rapidly:

29. $2 \times .33\frac{1}{3}$
30. $5 \times .16\frac{2}{3}$
31. $3 \times .12\frac{1}{2}$
32. $5 \times .12\frac{1}{2}$
33. $7 \times .12\frac{1}{2}$
34. $3 \times .8\frac{1}{3}$
35. $4 \times .12\frac{1}{2}$
36. $6 \times .12\frac{1}{2}$
37. $2 \times .15$
38. $4 \times .01\frac{1}{4}$
39. $3 \times .16\frac{2}{3}$
40. $4 \times .16\frac{2}{3}$

Memorize the following table:

$\frac{1}{2} = 50\%$	$\frac{1}{5} = 20\%$	$\frac{5}{6} = 83\frac{1}{3}\%$	$\frac{1}{12} = 8\frac{1}{3}\%$
$\frac{1}{3} = 33\frac{1}{3}\%$	$\frac{2}{5} = 40\%$	$\frac{1}{8} = 12\frac{1}{2}\%$	$\frac{5}{12} = 41\frac{2}{3}\%$
$\frac{2}{3} = 66\frac{2}{3}\%$	$\frac{3}{5} = 60\%$	$\frac{3}{8} = 37\frac{1}{2}\%$	$\frac{1}{16} = 6\frac{1}{4}\%$
$\frac{1}{4} = 25\%$	$\frac{4}{5} = 80\%$	$\frac{5}{8} = 62\frac{1}{2}\%$	$\frac{1}{20} = 5\%$
$\frac{3}{4} = 75\%$	$\frac{1}{6} = 16\frac{2}{3}\%$	$\frac{7}{8} = 87\frac{1}{2}\%$	$\frac{1}{25} = 4\%$

Name rapidly the fractional equivalents of the following per cents:

41. 50%	**46.** 20%	**51.** $37\frac{1}{2}\%$	**56.** 90%
42. $33\frac{1}{3}\%$	**47.** 40%	**52.** $62\frac{1}{2}\%$	**57.** $12\frac{1}{2}\%$
43. $66\frac{2}{3}\%$	**48.** 60%	**53.** $87\frac{1}{2}\%$	**58.** $16\frac{2}{3}\%$
44. 25%	**49.** 16%	**54.** 10%	**59.** 80%
45. 75%	**50.** $83\frac{1}{3}\%$	**55.** 30%	**60.** 70%

Write the equivalents of the following in decimals, thus: $1\% = .01$; $32\% = .32$; $\frac{1}{2}\% = .00\frac{1}{2}$; etc.

61. 1%	**67.** $\frac{5}{6}\%$	**73.** 50%	**79.** 13%
62. 32%	**68.** 3%	**74.** $\frac{2}{3}\%$	**80.** $13\frac{1}{3}\%$
63. $\frac{1}{2}\%$	**69.** 11%	**75.** 6%	**81.** $\frac{5}{8}\%$
64. 2%	**70.** $\frac{1}{4}\%$	**76.** $\frac{3}{4}\%$	**82.** 100%
65. $16\frac{1}{3}\%$	**71.** 4%	**77.** 7%	**83.** 123%
66. $\frac{1}{3}\%$	**72.** $43\frac{1}{2}\%$	**78.** $\frac{7}{8}\%$	**84.** 127%

Finding a given per cent of a number.

Recite the following thus: *Look at* "$66\frac{2}{3}\%$," *think* "$\frac{2}{3}$":

1. $66\frac{2}{3}\%$ of 18.
2. $33\frac{1}{3}\%$ of 90.
3. 50% of \$500.
4. 25% of \$2000.
5. 75% of 16 inches.
6. 20% of 100 yards
7. 40% of 60 feet.
8. 60% of 40 miles.
9. 80% of 75 gallons.
10. $16\frac{2}{3}\%$ of \$6000.
11. $83\frac{1}{3}\%$ of \$1200.
12. $37\frac{1}{2}\%$ of \$7200.
13. $12\frac{1}{2}\%$ of \$6400.
14. 75% of \$4800.
15. $66\frac{2}{3}\%$ of \$999.
16. 80% of 60 sheep.
17. 60% of 75 horses.
18. 40% of 90 miles.
19. $87\frac{1}{2}\%$ of \$160.
20. $62\frac{1}{2}\%$ of \$240.
21. $37\frac{1}{2}\%$ of \$880.
22. $12\frac{1}{2}\%$ of 24.

Written Work.

1. A man had 100 cows and sold 25% of them. How many did he sell?

25% = .25

100 cows

.25

500

200

25.00 number sold

As 25% = .25 we multiply 100 by .25. The result is 25, the number sold.

Find results decimally:

2. 50% of 750
3. 25% of 85.5
4. 40% of 8.75
5. 28% of 840
6. 32% of 1000
7. 75% of 980

8. John earns \$21.60 per month, and spends 75% for clothes. How much do his clothes cost him?

9. There are 780 pupils in school, and 40 % are males. How many are males?

10. If a man buys a horse for $150 and sells it at a profit of 20 %, how much does he gain?

In each of the preceding problems we have two terms, a *per cent* and a *whole* or a *mixed* number. The per cent in each problem is the multiplier, and is called the *rate*. The whole or the mixed number is the multiplicand, and is called the *base*. The product is called the *percentage*.

The **base** is that number of which some per cent is to be taken; as, 5 % of $200 (*base*).

The **rate** is the number of hundredths taken; as, 5% (*rate*) of 80 horses; that is, $\frac{5}{100}$ of 80 horses.

The **percentage** of a number is the result obtained by taking any per cent of it; as 10 % of 200 acres is $\frac{10}{100}$ of 200 acres, or 20 acres (*percentage*).

11. What is 75% of $5.12?

Multiplier		Multiplicand		Product
Rate		Base		Percentage
75 %	of	$5.12	=	( )

Decimal Method

75 % = .75.

$5.12 = base

.75 = rate

2560

3584

$3.8400 = percentage

Fractional Method

75 % = $\frac{3}{4}$

$\frac{3}{4}$ of $5.12 = $3.84

Study of Problem

What is the base? $5.12. What is the rate? 75%.

To what do the *base* and *rate* correspond in simple multiplication? *Multiplicand* and *multiplier*.

To what does *percentage* correspond in simple multiplication? *Product*.

How is the *product* found in simple multiplication? *Multiplier* × *multiplicand*.

How is the *percentage* found? *Rate* × *base*.

The **percentage of a number** *equals the product of the base by the rate.*

Find:

12. 6 % of $200.
13. 33⅓ % of 6 months.
14. 60 % of 30 days.
15. 1 % of 100 acres.
16. 5 % of 100 acres.
17. 8 % of 400.
18. 7 % of 400 horses.
19. 3⅓ % of 99.
20. 6 % of 150 lb.
21. 1½ % of $75.

22. 80 is the base, 25 % is the rate, find the percentage.

23. A house costs $2500, and the damage by fire is 8 %. Find the amount of the damage.

24. John owes his tailor $80, and pays 37½ %. How much does he still owe him?

25. Mary spells 90 % of 80 words correctly. How many does she miss?

26. A boy buys apples at $1 per bushel, and sells them at a profit of 20 %. How much profit is that per bushel?

27. 6¼ % of 3680 equals what number?

28. A man buys a farm for $2500, and sells it for 25 % more than it cost him. For how much does he sell the farm?

29. A man earns $180 per month, and puts 33⅓ % of it in the savings bank. What is his deposit each month?

30. If 37½ % of a man's farm is in timber, and the total area is 240 acres, how much timber land has he?

31. A teacher who earned $1200 a year spent 66⅔ % of her salary. How much did she save?

32. Mr. Scott's horse is valued at $250 and Mr. Hill's at 60 % of this. What is the value of Mr. Hill's horse?

33. The population of a town of 9672 inhabitants increased 12½ % in a year. What was the increase in population?

Finding what per cent one number is of another number.

1. What % of 8 is 4?

4 is ½ of 8, or 50% of 8.

What % of:

2. 10 is 5?	7. 12 is 9?	12. $12\frac{1}{2}$ is $6\frac{1}{4}$?
3. 12 is 6?	8. 20 is 15?	13. $27\frac{1}{2}$ is $5\frac{1}{2}$?
4. 20 is 5?	9. 25 is 5?	14. $22\frac{1}{2}$ is $7\frac{1}{2}$?
5. 12 is 4?	10. 40 is 8?	15. $17\frac{1}{2}$ is $3\frac{1}{2}$?
6. 16 is 8?	11. 30 is 10?	16. $19\frac{1}{2}$ is $6\frac{1}{2}$?

Written Work

1. What per cent of 40 is 10?

$10 \div 40 = \frac{10}{40} = .25$ or 25%

Test: $.25 \times 40 = 10$.

Since the percentage equals the base *multiplied* by the rate, the rate must equal the percentage *divided* by the base. $10 \div 40 = \frac{10}{40} = .25$ or 25%.

The **rate** *equals the percentage divided by the base.*

What per cent of:

2. 50 is 25?	5. 100 is 60?	8. 150 is 25?
3. 80 is 20?	6. 125 is 25?	9. 160 is 60?
4. 60 is 12?	7. 240 is 60?	10. 320 is 25.6?

11. Frank's salary for a month is $75 and Samuel's is $25. What per cent of Frank's salary is Samuel's?

12. I earn $600 a year and spend $300. What per cent of my money do I spend?

13. A farmer who raised 350 bushels of potatoes marketed only 280 bushels. What per cent of his potatoes did he market?

Finding a number when a per cent of it is given.

1. If $\frac{1}{2}$ of a number is 5, what is the number?
2. If 50 % of a number is 5, what is the number?

5 is $\frac{50}{100}$ or $\frac{1}{2}$ of 10.

Find the number of which:

3. 10 is 25 %
4. 25 is 50 %
5. 20 is $33\frac{1}{3}$ %
6. 15 is 60 %
7. 60 is 75 %
8. 40 is 20 %
9. 10 is 5 %
10. 50 is $12\frac{1}{2}$ %
11. 30 is $66\frac{2}{3}$ %
12. 50 is $62\frac{1}{2}$ %

Written Work

1. Find the number if 25 % of it is 128.

$128 \div .25 = 512$

Test: $.25 \times 512 = 128$

Since the percentage equals the base *multiplied* by the rate, the base equals the percentage *divided* by the rate. $128 \div .25 = 512$, the number.

The **base** *equals the percentage divided by the rate.*

Find the number if:

2. 10 % of it is \$35
3. 20 % of it is \$38
4. 30 % of it is \$45
5. $12\frac{1}{2}$ % of it is \$56
6. $33\frac{1}{3}$ % of it is \$65
7. $37\frac{1}{2}$ % of it is \$72
8. $62\frac{1}{2}$ % of it is \$75
9. 75 % of it is \$96
10. 60 % of it is \$84
11. 8 % of it is \$24
12. 35 % of it is \$42
13. $87\frac{1}{2}$ % of it is \$70

14. A boy paid \$3 or 20 % of his month's pay for a pair of shoes. How much does he earn in a month?

15. A teacher pays \$20 per month for board, and this is 40 % of his salary. What is his monthly salary?

GAIN AND LOSS

When people buy or sell, they always reckon their *profit* or their *loss* on what a thing *costs*.

1. A boy buys bananas at 30 cents per dozen and sells them at $33\frac{1}{3}\%$ gain. For how much does he sell the bananas per dozen?

The gain is $\frac{1}{3}$ of 30¢, or 10¢ per dozen. Therefore the selling price is 30¢ + 10¢, or 40¢ per dozen.

2. Bread is bought at 5 cents a loaf and sold at 6 cents. What is the gain per cent?

The gain on 1 loaf is 1¢. 1¢ = $\frac{1}{5}$ of 5¢ = 20% gain.

Find the gain or loss, when goods are bought at:

3. \$40 and sold at 20% profit; at 20% loss.

4. \$80 and sold at 25% profit; at $12\frac{1}{2}\%$ loss.

Find the gain or loss per cent when goods are sold at:

5. \$40 that cost \$30; that cost \$50.

6. \$60 that cost \$75; that cost \$50.

7. If a dealer buys shoes at \$18 a dozen pairs and sells them at 50% gain, what is the retail price?

8. A boy buys oranges at 40¢ a dozen and sells them at 5¢ apiece. What per cent does he gain?

9. A man buys a horse for \$150 and sells it for \$175. What per cent does he gain?

10. A boy buys a pony for \$50 and sells it for \$62.50. What per cent does he gain?

11. A man sells a city lot that cost him \$1200 at a loss of 15%. Find the amount he loses.

12. A merchant buys goods at \$.75 per yard and retails them at a profit of 40%. Find the gain on 36 yards.

COMMISSION

An **agent** is a person who transacts business for another.

Commission is the sum charged by an agent or commission merchant for his services.

The **net proceeds** is the sum left after the commission and other expenses have been paid.

1. A real estate agent sold a house for $5000, retaining 5% of this sum for his services. How much did he receive? How much did the owner receive?

$5000 = selling price.
.05 = rate charged by the agent.
$250.00 = amount charged by the agent.
$5000 − 250 = $4750, amount received by the owner.

A commission merchant made the following sales. Find his commission for each day at 5%.

2. Monday, $1800
3. Tuesday, $1594
4. Wednesday, $1954
5. Thursday, $1400.80
6. Friday, $1528
7. Saturday, $2370.60

8. Find his total commission for the week.

9. A real estate agent sells a house and lot for $6750, charging 2% commission. Find his commission and the net proceeds.

10. A traveling salesman sold $50,000 worth of goods in a year at a commission of 8%. If his expenses for the year were $2200, how much had he left?

11. An agent rents 12 houses at $40 per month. If he receives 5% for collecting the rents, how much is remitted to the owners each month?

COMMERCIAL DISCOUNT

Wholesale merchants and manufacturers usually publish printed price lists of their goods. The prices in these lists are higher than the wholesale prices and are subject to deductions called **trade discounts** or **commercial discounts.**

NOTE. — A **discount** is any deduction from a fixed price.

Sometimes several discounts are allowed. The first is a discount from the list price; the second, a discount from the remainder, etc.

The **net price** is the price after deduction of all discounts.

Find the selling price of goods marked :

1. \$15, less 20 %.
2. \$20, less 40 %.
3. \$6, less 50 %.
4. \$25, less 20 %.
5. \$7.50, less 20 %.
6. \$12.50, less 40 %.
7. \$40, less 60 %.
8. \$48, less 25 %.
9. \$6.80, less 25 %.
10. \$4.50, less $33\frac{1}{3}$ %.
11. \$9.60, less $16\frac{2}{3}$ %.
12. \$4.80, less $37\frac{1}{2}$ %.

Written Work

Find the selling price of goods marked :

1. \$168.75, less 25 %.
2. \$1374, less $16\frac{2}{3}$ %.
3. \$1872, less $33\frac{1}{3}$ %.
4. \$278.40, less $37\frac{1}{2}$ %.
5. \$3030 less 40 %.
6. \$225.65, less 20 %.
7. \$875.50, less 30 %.
8. \$278.90, less 10 %.
9. \$2378.50, less 4 %.
10. \$6775.20, less 5 %.

Find the cost of :

11. Discount, 20 %
    60 readers @ \$.40
    150 geographies @ \$1
    78 grammars @ \$.60

12. Discount, 4 %
    160 lb. rice @ \$.06
    300 lb. sugar @ $\$.04\frac{1}{2}$
    200 lb. coffee @ \$.16

13. Find the net price of a bill of goods for \$ 75.40, trade discounts 20 %, 10 %.

List price,	\$ 75.40
Less 20 %,	15.08
First remainder,	60.32
Less 10 %,	6.03
Net price,	\$ 54.29

Observe that the *second* discount is reckoned on the *first* remainder. As there are only two discounts, the second remainder is the net price.

Find the net price of articles listed at:

14. \$ 400, less 20 %, 10 %.

15. 375.50, less 25 %, 5 %.

16. 290.80, less 40 %, 10 %.

17. \$ 10.75, less 40 %, 5 %.

18. \$ 6.80, less 25 %, 10 %.

19. \$ 12.75, less $33\frac{1}{3}$ %, 10 %.

Find the net price of the following bills of goods:

20. 36 dozen boys' caps @ \$ 6, discounts 25 %, 20 %.

21. 50 buggies @ \$ 120, discounts 20 %, 15 %.

22. 75 sets harness @ \$ 40, discounts 30 %, 10 %.

23. 25 grain drills @ \$ 95, discounts 40 %, 5 %.

24. 12 rubber hose, each 50 feet long at 15 ¢ per foot, discounts 30 %, 15 %.

25. Mr. Austin buys a wagon listed at \$ 95, less 20 %, 15 %. Find the amount paid for the wagon.

26. A merchant buys 12 stoves listed at \$ 45, less 40 %, 10 %. Find the net amount of the bill. Compare this with the net amount of the bill with only one discount of 50 %.

27. A hotel keeper buys 675 yards of carpet at \$ 1.25, less 20 %, 5 %. Find the cost of the carpet.

28. Compare the net price of an article listed at \$ 500, discounts of 20 %, 10 % with the net price of a similar article listed at \$ 5.00, discounts of 10 %, 20 %.

Discounting Bills.

Merchants and manufacturers frequently offer an extra discount from their bills if cash is paid at the time of purchase or within a certain time. This is called a **cash discount.** Cash discounts are always mentioned in connection with the *terms of payment.*

1. Find the net cash price of the sectional bookcases as given in the following bill at $33\frac{1}{3}\%$ and 10% trade discounts. Terms: 60 days net or 5% cash.

PITTSBURG, PA., *Jan. 4*, 1907.

Messrs. *Geo. H. Alexander & Co.*,
300 Wood Street.

Bought of THE KENSINGTON MFG. CO.
GRAND RAPIDS, MICH.

TERMS: 60 days net, or 5% cash

Jan. 4	*'07*	*12*	*Bookcase Sections* $28	$14	40	
		12	*" Tops* $21	9		
		6	*" Bases* $25	27		
				50	40	
			Less $\frac{1}{3}$, $\frac{1}{10}$,	20	16	
			Net price,	30	24	
			Cash, less 5%	1	51	$28 73
			Received payment,			
			Kensington Mfg. Co.			
			Per R.			

The sign #, when placed before a number, is read "number." Thus # 12 is read "number 12."

Make out bills for the following:

2. The Packard Hardware Co. bought for cash from Jas. M. Armstrong Co., Chicago, Ill., 4 doz. Acme lawn mowers @ $10, 50 lb. lawn seed @ 15¢, 2½ doz. brushes @ 40¢. Trade discounts: 20%, 10%. Terms: 10 days net, or 2% cash.

3. Jamison and Redmond, South Bend, Ind., bought, for cash, from the Acme Buggy Co., Cincinnati, O., 72 buggies @ $105, 50 sets harness @ $45, 15 sleighs @ $60, 40 robes @ $20. Trade discounts: 30%, 15%. Terms: 30 days net, or 3% cash.

4. James Cubbison, Greenville, O., buys for cash from Arbuthnot, Stevenson & Co., Pittsburg, Pa., 5 doz. handkerchiefs @ $3.60; 5 bolts muslin, 40 yd. each, @ 8¢; 5 bolts prints, 42 yd., @ 7¢. Trade discount: 33⅓%. Terms: 10 days net, or 2% cash.

5. S. H. Gardner Co., piano dealers, Detroit, Mich., order from the Harmonic Piano Co., Chicago, Ill., 2 Harmonic pianos #266 @ $600, less 40%, 10% trade discount. Terms: 90 days net, or 10% off 10 days. Find the net cash price. Find the net price if paid in 30 days.

6. M. L. Smith, tailor, Brockton, Mass., orders from Bender & Co., New York, importers, 3 pieces suiting, 22 yd. each, @ $3.15. Terms: 30 days net, or 2% off 10 days. Make out and receipt bill if paid within 10 days.

INTEREST

1. Mr. Johnston pays the liveryman $6 for the use of a horse and buggy for two days. What does he get in exchange for the $6?

2. Mr. Daniels pays $6 for the right to pasture his cow in a field for two months. What does he get in exchange for the $6?

3. Mr. Watson pays $6 for the use of $100 for one year. What does he get in exchange for the $6?

4. In the first two examples money is paid for the use of something that is not money. For what does Mr. Watson pay the money in the last example?

Interest is compensation for the use of money. Interest corresponds to the *percentage* in percentage.

5. How much does Mr. Watson pay for the use of the money? What is the $6 called?

6. On what is the interest reckoned? The $100 is called the *principal.*

The **principal** is the sum on which the interest is paid. The principal corresponds to the *base* in percentage.

The **rate of interest** is a certain number of hundredths of the principal paid for the use of the principal for *one year.*

Time is always a factor in *interest.* **Interest,** then, is the product of three factors: **principal, rate,** and **time.**

The **amount** is the sum of the principal and the interest.

Interest for Years and Months

1. What part of a year are 6 months? 4 months? 3 months? 2 months? 1 month?

2. If the interest for a year is $100, what should it be for 6 months? for 4 months? for 3 months? for 2 months? for 1 month?

Written Work

1. What is the interest on $200 for $2\frac{1}{2}$ years at 6%?

$200 principal
.06 rate
$12.00 interest for one year
$2\frac{1}{2}$
$30.00 interest for $2\frac{1}{2}$ years

The interest for 1 year is .06 of the principal, or $12. The interest for $2\frac{1}{2}$ years is $2\frac{1}{2} \times \$12$, or $30.

Multiply the principal by the rate and the product by the number of years.

The year is usually considered as 360 days, that is, 12 months of 30 days each.

Find the interest on:

2. $300 at 5% for 1 year.
3. $800 at 8% for 2 years.
4. $150 at $6\frac{1}{2}$% for 3 years.
5. $700 at $4\frac{1}{2}$% for 4 years.

Find the interest of:

6. $250 for $1\frac{1}{2}$ years at 4%.
7. $75 for 2 years at 8%.
8. $100 for $3\frac{3}{8}$ years at 7%.
9. $80 for $4\frac{1}{2}$ years at 5%.
10. $40 for $2\frac{1}{2}$ years at $6\frac{1}{2}$%.
11. $500 for $2\frac{1}{3}$ years at $4\frac{1}{2}$%.
12. $960 for 9 mo. at 6%.
13. $900 for $2\frac{3}{4}$ years at 7%.
14. $654 for $\frac{3}{4}$ year at 6%.
15. $220 for $\frac{7}{8}$ year at 8%.

Find the interest at 6% on :

16. $100 for 6 months.
17. $500 for 4 months.
18. $150 for 2 yr. 2 mo.
19. $624 for 120 da.
20. $170 for 8 mo.
21. $355 for 130 da.

Interest for Years, Months, and Days

1. What part of a month (30 days) are 15 days? 12 days? 20 days? 3 days? what part is 1 day?

2. If the interest for 1 year is $360, what is the interest for 1 month? If the interest for 1 month is $30, what is the interest for 1 day? for 15 days? for 12 days?

Written Work

1. Find the amount of $200 at 6% interest for 2 yr. 7 mo. 12 da.

	Principal =	$200	
	Rate =	.06	
	Int. for 1 yr. =	$12.00	
Int. for 2 yr.	= 2 × $12.00	or	$24.00
Int. for 7 mo.	= $\frac{7}{12}$ of $12.00	or	7.00
Int. for 12 da.	= $\frac{1}{30}$ or $\frac{1}{3}$ of $1.00	or	.40
Int. for 2 yr. 7 mo. 12 da. =			$31.40
	Principal =		$200.00
Amount for 2 yr. 7 mo. 12 da. =			$231.40

Study of Problem

a. What is the *first* step in the work? the *second* step?

b. How do we find the interest for 1 month? for 7 months? for 12 days?

c. What new term is introduced in interest? For what length of time is *rate* of interest always considered?

Find the interest and amount of:

2. $300 for 3 yr. 6 mo. at 6%.
3. $250 for 2 yr. 4 mo. at 7%.
4. $160 for 4 yr. 3 mo. at 5%.
5. $50 for 1 yr. 8 mo. at 5%.
6. $800 for 3 yr. 2 mo. at 6%.
7. $50.80 for 9 mo. at 10%.
8. $16 for 8 mo. at 6%.
9. $75 for 8 mo. at 6%.
10. $420 for 10 mo. at 10%.
11. $40.50 for 1 yr. 1 mo. at 6%.
12. $300.40 for 5 mo. at 7%.
13. $100 for 7 mo. at 7%.
14. $500 for 11 mo. at 6%.
15. $1000 for 1 mo. at 6%.
16. $60.60 for 8 mo. at 8%.

Find the interest and amount of:

17. $250 at 8% for 3 yr. 5 mo. 20 da.
18. $75.80 at 5% for 4 yr. 1 mo. 16 da.
19. $1500 at 6% for 2 yr. 9 mo. 15 da.
20. $125.50 at 4% for 4 yr. 11 mo. 12 da.
21. $1140 at $5\frac{1}{2}$% for 4 yr. 8 mo. 24 da.
22. $912.60 at 5% for 2 yr. 10 mo. 11 da.
23. $3209 at 6% for 3 yr. 7 mo. 21 da.
24. $634.50 at 8% for 11 mo. 12 da.

25. Henry Boydson borrows $275 Sept. 1, 1906, at 6% interest, and settles the note Jan. 1, 1908. Find the amount of the note at settlement.

The one dollar six per cent method of finding interest.

1. What is the interest on $1 at 6% for 1 year?

Interest on 1 dollar for 1 *month* equals $\frac{1}{12}$ of $.06 or $\frac{1}{2}$¢.

Interest on 1 dollar for 1 *day* equals $\frac{1}{30}$ of $\frac{1}{2}$¢ = $\frac{1}{60}$¢ = $.000$\frac{1}{6}$.

Since the interest on 1 dollar for 1 month is $\frac{1}{2}$¢ and for 1 day $\frac{1}{6}$ mill, *change the years and months to months. The interest on each dollar will be $\frac{1}{2}$ as many cents as there are months and $\frac{1}{6}$ as many mills as there are days. Multiply the sum by a number equal to the number of dollars.*

The interest at any other rate may be found as follows:

1% = $\frac{1}{6}$ of 6%	5% = 6% − $\frac{1}{6}$ of 6%
2% = $\frac{1}{3}$ of 6%	7% = 6% + $\frac{1}{6}$ of 6%
3% = $\frac{1}{2}$ of 6%	8% = 6% + $\frac{1}{3}$ of 6%
4% = $\frac{2}{3}$ of 6%	9% = 6% + $\frac{1}{2}$ of 6%

Write the interest on 1 dollar at 6% for:

2. 12 mo.
3. 8 mo.
4. 16 mo.
5. 8 mo. 18 da.
6. 1 yr. 2 mo. 12 da.
7. 1 yr. 6 mo. 15 da.
8. 1 yr. 9 mo. 12 da.
9. 2 yr. 4 mo. 18 da.

Written Work

1. Find the interest and amount of $300 for 1 year 10 months 19 days at 6%; at 8%.

1 yr. 10 mo. = 22 months.

Int. on $1 at 6% for 22 mo. = $.11

Int. on $1 at 6% for 19 da. = $.003$\frac{1}{6}$

Int. on $1 at 6% for 22 mo. 19 da. = $.113$\frac{1}{6}$

Int. on $300 at 6% for 22 mo. 19 da. = 300 × $.113$\frac{1}{6}$

or $33.95 (to the nearest cent).

$33.95 int. + $300 prin. = $333.95, amount.

At 8% the interest will be $33.95 + ($\frac{1}{3}$ of $33.95), or $45.27, and the amount $345.27.

Find the interest and amount of :

2. $660 at 6% for 1 yr. 2 mo. 12 da.

3. $457.75 at 6% for 2 yr. 8 mo. 18 da.

4. $675 at 6% for 1 yr. 4 mo. 15 da.

5. $1200 at 6% for 2 yr. 8 mo. 21 da.

6. $84.50 at 6% for 3 yr. 3 mo. 24 da.

7. $90.75 at 6% for 2 yr. 6 mo. 6 da.

8. Find the amount necessary to pay a loan of $250 at 6% from Jan. 4, 1907, to Jan. 25, 1909.

9. Find the interest on $375 borrowed Sept. 1, 1906, and paid March 16, 1908.

10. Henry Boydson borrows Jan. 4, 1907, from James Anderson $375, agreeing to pay the money April 1, 1909, with interest at 6%. Find the amount due James Anderson April 1, 1909. The following is the form of the written promise.

Promissory Note

$375.00 *Augusta, Me., Jan. 4, 1907.*

April 1, 1909, I promise to pay to the order of

James Anderson

Three hundred seventy-five ~~~~~ *Dollars*

with interest at 6%.

Value received, *Henry Boydson.*

A **promissory note** is a written agreement made by one person to pay to another a sum of money at a certain time.

SAVINGS ACCOUNTS FOR BOYS AND GIRLS

Interest on savings accounts is usually computed at from 2 % to 4% annually, and is *payable the first of each January and July*. If not drawn when due, the interest or **dividend**, as it is sometimes called, is added to the deposit. For example: If John deposited $10 Jan. 1, his amount in the bank July 1 would be $10 plus the interest on $10 for 6 months at 4%, or $10.20. Since interest on savings accounts is not computed on parts of a dollar, the interest for the next 6 months would be computed on $10 only.

Interest on savings accounts is frequently calculated *from the 1st and 15th of each month succeeding the several deposits.* Thus, $10 deposited the 1st of any month would draw interest from date; but $10 deposited the 2d of any month would draw interest from the 15th, and money deposited on the 15th would draw interest from the 1st of the next month.

Thirty days are reckoned as a month.

1. On Jan. 1, 1907, John deposited $10 in the savings bank at 4% interest. On March 25 he deposited $5. What was John's amount in bank July 1, 1907?

$10 prin. Jan. 1, 1907.
.04 rate
2)$.40 interest for 1 yr.
$.20 interest for 6 mo.

$5 prin. Mar. 25, 1907.
.04 rate
$ .20 interest for 1 yr.
Interest from April 1st, to July 1, — 3 mo., or ¼ of a year.
4) .20 interest for 1 yr.
.05 interest for 3 mo.

$10. first deposit.
.20 interest on first deposit.
5.00 second deposit.
.05 interest on second deposit.
$15.25 total amount in bank July 1, 1907.

2. Mary Thompson deposits $5.00 in the American Savings Bank Jan. 1, 1908. March 1, 1908, she makes another deposit of $6. Find total amount in bank Jan. 1, 1908.

3. Frank makes the following deposits: Jan. 1, 1906, $10, Feb. 10, 1906, $15. Find total Jan. 1, 1907.

NOTE. — Every boy or girl in school should be encouraged to keep a savings bank account, however small.

4. Find the interest July 1, 1907, on a deposit of $100 made July 1, 1906, at 4% interest, payable January 1 and July 1.

5. The total deposits of the pupils in a certain school Jan. 1, 1906, are $395.75. Find the amount of these deposits July 1, 1907, at 4% interest, dividends being payable each January 1 and July 1.

6. Henry's parents put $100 in a savings bank Jan. 1, 1907, at 4% interest, dividends payable each January 1 and July 1. Find the amount of this deposit July 1, 1909.

7. A newsboy put $50 in a savings bank Jan. 1, 1906, and $50 the first of each January and July following. Find the amount in bank July 2, 1908, at 4% interest, dividends payable each January 1 and July 1.

8. The Jackson Street school has in the savings bank $508.70 Jan. 1, 1907, and makes a deposit April 1, 1907, of $230.50. Find the amount of these two deposits July 1, 1908, at 4% interest, dividends payable January 1 and July 1.

9. A school teacher places $30 in a savings bank the first of each month beginning with October and ending with July 1. Find the amount in bank after the last deposit, at 4% interest, dividends payable January 1 and July 1.

REVIEW OF PERCENTAGE AND INTEREST

1. A boy has $30 in a savings bank and deposits a sum equal to 10% more. What is his total deposit?

2. Mr. James's salary is $1200 per year and he saves $33\frac{1}{3}\%$ of it. How much does he spend?

3. Hats that cost $36 per dozen are sold at $2 apiece. Find the loss per cent.

4. My deposit in the bank on Thursday was $80; on Saturday my balance was $50. What per cent of my money did I draw out?

5. A boy spends $8 for an overcoat and $37\frac{1}{2}\%$ of that sum for shoes. How much does he spend for shoes?

6. In a school of 45 pupils, $33\frac{1}{3}\%$ of the pupils are boys. What is the number of girls?

7. My deposit January 1 in the savings bank is $25. What is the interest on it one year later at 4% interest, dividends payable the first of each January and July?

8. Find 25% of .05; of .5; of 5.5; of .25.

9. John earns $50 during his vacation and Margaret 25% as much as John. How much does Margaret earn?

10. If a man spends one fourth of his salary for board and clothes, what per cent of his salary does he save?

11. What per cent of $80 is $60?

12. A man buys a horse for $80 and sells it for $100. Find the per cent gained.

13. A house is insured for $1200, and the insurance company agrees to pay $120 for damage done to it by fire; what per cent of the total insurance is paid?

14. A farmer sold a horse that cost him $80 at a loss of 20%. Find the selling price.

15. What is the interest of $150 for $2\frac{1}{2}$ years at 6%?

16. It cost $2.50 to repair my watch valued at $20. What per cent of the value did I pay for repairs?

17. Find the interest on $100 for 60 days at 5%.

18. My father borrows $75 from his neighbor and promises to pay it in 4 months at 6%. Find the amount my father must pay at the end of four months.

19. A huckster buys eggs at $.20 per dozen. For how much per dozen must he sell them to gain 20%?

20. If I borrow $50 from Mr. James for 6 months at 6%, how much interest must I pay him?

21. A grocer sold flour last week at $1.20 per sack and this week at 10% advance on last week's selling price. Find the price of flour per sack this week.

22. A huckster buys 150 dozen eggs at $.20 per dozen and sells them to a merchant at a gain of 25%. The merchant sells them at a gain of 20%. How much does the merchant receive for the eggs?

23. If I buy cloth at $1.50 a yard, for how much must I sell it to gain $33\frac{1}{3}$%?

24. A man sold a horse for $120 that cost him $150. What % did he gain or lose?

25. A boy deposits $50 the first of each year in a savings bank that pays 4% interest, payable the first of each January and July. What will be the total amount in bank after he has made two deposits?

26. A grocer buys goods to the amount of $1200, 10% off for cash. He sells them for $1500 cash. How much does he gain?

27. A mother took two boys and a girl to a store to buy clothes. The first boy's suit cost $10 less 10% for cash. The second boy's suit cost $66\frac{2}{3}\%$ of the cash sale of the first boy's suit. The girl's coat cost $33\frac{1}{3}\%$ of the money paid for both boys' suits. How much did the mother pay for the children's clothes?

28. Find 25% of 200, and divide the result by $.00\frac{1}{4}$.

29. A farmer sold 425 sheep, which were 34% of all he had. How many sheep did he have?

30. A man buys a house and lot for $5000. It costs every year $25 for repairs and $50 for taxes and insurance. He rents the house for 8% of its cost. How much has he left after paying expenses?

31. A coal dealer bought 300 tons of coal for $600. The freight, storage, and delivery cost $33\frac{1}{3}\%$ of the cost of the coal. What was the retail price per ton if he sold it at a gain of $12\frac{1}{2}\%$?

32. If 40% of a man's interest in a store is valued at $14,000, what is his interest?

33. A real estate agent purchased a house for $1250. For how much per month must he rent the house to make 6% after paying each year $18 for taxes and insurance and $15 for repairs?

34. A merchant has invested in his building and store $6000. After paying all the expenses he finds he has cleared for the year $1200. What per cent does he make on the cost of the building and store?

RECEIPTS AND CHECKS

John Watson pays James Adams $35.50 for work for one month, and asks Mr. Adams for a receipt. Fill out this receipt to show that the money was paid by Mr. Watson and received by Mr. Adams.

Rochester, N. Y., June 1, 1907.

Received from _____

_____ *Dollars*

for _____

$ _____ _____

1. What must every receipt show? (see p. 99).

2. Write the receipt your grocer would give you in payment of $18.50 on account by your father or mother.

3. Your school district pays the National Book Company, New York, $ 25.75 for school books on Sept. 15, 1907. Make out the receipt of the National Book Company.

4. Henry Smith received $ 3.65 from James Brown for 3 months' water rent. Make out a receipt for the amount.

5. Ralph Taylor pays H. W. Henderson $5 for a month's tuition. Write the receipt Ralph Taylor should receive.

6. Write a receipt for $75 which Nelson Page paid Edgar Poe for balance due on a buggy.

7. Make out and receipt the bill for the following articles bought by James Thomas from Jos. Horne & Co.:

3 shirts	@	$1.75	2 neckties	@	$.75
6 collars	@	.20	4 pairs cuffs	@	.20

8. Presuming that you are a collector for the *Gazette-Times*, Pittsburg, Pa., make out a receipt to a subscriber who has paid you $2.60 in full of account.

A **check** is an order on a bank where a person keeps a deposit, ordering the bank to pay money.

No. 875 STUB *$67.00* *Jan. 10, '07* *To James Ward* *For Labor*	*No. 875* *Seattle, Wash., Jan. 10, 1907.* The Yukon National Bank of Seattle. *Pay to the order of James Ward $67.00* *Sixty-seven 00/100 ~~~~~ Dollars.* *W. J. Moore.*

Study of Check

1. Name the different things stated in this check.

2. Observe that this check is payable to the *order of* James Ward. He orders it paid by writing his name across the back of it. This is called **indorsing** the check.

3. Write the check your father would give your teacher in payment of $3.50 for September tuition.

4. Emil Smith borrows from Joseph McLean $240 to attend school and pays the same in 2 yr. 4 mo. 18 da. at 6%. Write the amount of the check that would pay the note.

GENERAL REVIEW

1. The remainder is 92,568 and the minuend is 202,660. Find the subtrahend.

2. The dividend is 364,450 and the quotient is 9850. What is the divisor?

3. Add 3.5, .035, 45.006, and 2.06.

4. Write decimally twenty-five and sixty-one thousandths; one hundred twenty-five and five tenths; and three hundred and two ten-thousandths.

5. What number multiplied by one hundred seventy-nine is equal to 848,818?

6. From 2.0011 take 1.9892.

7. Explain the difference between $\frac{1}{5}\%$ and $\frac{1}{5}$.

8. Add $\frac{1}{2}$, $2\frac{3}{4}$, $\frac{1}{3}$, $\frac{23}{36}$, and $4\frac{5}{6}$.

9. Find 1%, $\frac{1}{5}$, $\frac{1}{5}\%$, $\frac{1}{20}$, $\frac{1}{20}\%$, 50%, and $\frac{1}{10}$ of 100.

10. The multiplicand is 1325 and the multiplier is .0416. What is the product?

11. If 38 dozen eggs cost $11.96, what is the cost per dozen?

12. A building is 46 ft. 3 in. wide, and twice as long as wide. Find the distance around the building.

13. From 86 miles and 3 inches, take 46 miles and 8 inches.

14. A man and his son together earn \$72 per month. If the man's earnings in 6 months amount to \$300, how much are the son's earnings in the same length of time?

15. A man bought 48 head of cattle, at \$36 per head, and sold them at a gain of 25%. What was the total amount received for the cattle?

16. Find the interest on \$370.50 for 4 yr. 8 mo. at 6%.

17. Divide $48\frac{4}{5}$ by $21\frac{4}{5}$.

18. Divide .65 by 6.5.

19. Reduce $187\frac{1}{2}$ rods to the fraction of a mile.

20. How much will it cost to ship a car load of wheat containing 42,000 lb. from Fargo, N.D., to Chicago, Ill., if the freight rate is \$.06 per bushel?

21. A train leaves Chicago at 8:15 A.M., and arrives at Pittsburg at 8:20 P.M. The distance is 468 miles. Find the number of miles per hour the train travels.

22. The steel rails on the Bessemer railroad weigh 100 pounds to the foot. Find the number of tons necessary to lay 5 rods of single track.

23. How much does an architect receive, at $4\frac{1}{2}$%, for the plans of a house that cost \$8350?

24. A man's salary is \$150 per month. He spends 40% of it for clothing and other expenses. How much does he save in a year?

25. A man purchases 80 acres of land at \$6400, and sells them at 25% gain. How much does he receive per acre?

26. A real estate man buys a house for \$2800 and sells it for \$3200. What is his gain per cent?

27. What is the cost of a house on which 25% is gained when it is sold at a profit of \$2500?

28. A town lot is 43 ft. 3 in. wide and 120 ft. deep. How much is it worth at 75 ¢ per square foot?

29. A western farmer harvests 8960 bu. wheat from a field 320 rd. long and 160 rd. wide. If he sells the wheat at 60 ¢ per bushel, how much does he realize from each acre?

30. In 1 hour 20 minutes and 40 seconds, a train travels 60 miles. At that rate how long would the train be in traveling 1200 miles?

31. The average wages of a steel mill employing 3000 men are $2.50 per day. If a 10 % reduction in wages is made, how much per day will the company's pay roll be reduced?

32. In a certain class the salary of the teacher for a year is $500. The books and supplies cost $90.65; fuel, $40; repairs and other expenses, $75.30. There are 35 pupils in the class. Find the average cost per pupil for the year.

33. Reduce to improper fractions: 6.25; 3.375; $4.66\frac{2}{3}$; and 2.05.

34. If it costs $72 to carpet a room 18 ft. long and 18 ft. wide, how much will it cost to carpet a room 36 ft. long and 36 ft. wide, with the same quality of carpet?

35. Mt. Rainier is 14,363 ft. high. Reduce the height to miles and the fractions of a mile.

36. How many cubic inches are there in a bin 9 ft. 7 in. long by 8 ft. 3 in. wide and 4 ft. 9 in. deep? how many cubic feet?

37. A grocer bought 225 bu. apples at $.50 per bushel. He sold 150 bu. at $.75 per bushel. The remainder, which were damaged, he sold at $.40 per bushel. Did he gain or lose and what per cent?

38. What is $33\frac{1}{3}\%$ of 24? of \$4.80? of \$62.50?

39. A piece of land 30 rods wide and 480 rods long was sold at \$62.50 per acre. Find the amount of the sale.

40. Time, 3 months; rate of interest, 5%; money borrowed, \$100. Find amount to be paid.

41. If $\frac{2}{3}$ of a bushel of potatoes cost \$.40, how much will $7\frac{1}{2}$ bu. cost?

42. A piece of land 40 rods long in the form of a rectangle contains 5 acres. Find its width in rods.

43. A farmer sold $12\frac{3}{4}$ acres of land at $\$55\frac{1}{2}$ per acre. How much did he receive for the land?

44. Houser Brothers sold the following bill of goods to William Pool:

12 lb. sugar	@ \$.06½
10 cans tomatoes	@ \$.15
6 lb. rice	@ \$.07½
11 lb. prunes	@ \$.07½
2 pair boots	@ \$3.50
1 overcoat	@ \$13.50
1 pair shoes	@ \$4.00

Mr. Pool at the same time sold Houser Brothers:

85 bu. potatoes	@ \$.65
50 bu. corn	@ \$.42½
16 lb. butter	@ \$.24
10 lb. butter	@ \$.28

Houser Brothers gave Mr. Pool the balance in cash. Make out the account.

45. A painter worked $17\frac{1}{2}$ days. After spending $\frac{4}{7}$ of his wages for board he had \$15 left. Find his daily wages.

46. I owe Frank Morrison, the grocer, $32.50 and pay him $23.75. Write the receipt that Mr. Morrison should give me.

NOTE. — When a debt is not paid in full, the receipt should read "On account."

47. A cellar 24 ft. by 32 ft. is to be excavated to an average depth of $5\frac{1}{2}$ ft. Find the number of cubic yards to be removed.

48. Express $22\frac{1}{2}$ yards as rods, feet, and inches.

49. The width of a rectangle is 20 rods and the area is 560 square rods. Find the length.

50. What is the difference between a square and a rectangle?

51. Give the rule for finding percentage. On what is gain or loss always reckoned?

52. A man's farm and personal property cost $5600. The first year he cleared $12\frac{1}{2}\%$ of the money invested. The second year, on account of floods, he lost 5% of the cost of the property. How much was his gain in the two years?

53. Of a bill of $155 sent to a collector, 80% was collected and the collector retained $12.40. What per cent did he charge for collecting?

54. A boy receives $1.20 per day and a man $2.50 per day. How long will it take the boy to earn as much as the man can earn in 30 days?

55. The perimeter of a rectangle is 72 rods. The width is 12 rods. Find the length.

56. Estimating that 300 cu. ft. of air is required for each pupil, how many pupils, including the teacher, should occupy a room 40 ft. long, 30 ft. wide, and 12 ft. high?

57. Divide one thousand and one thousandth by one thousand one thousandths.

58. What is the interest on $375 for 270 days at 6%?

59. Reduce $\frac{5}{8}$ of a mile to lower denominations.

60. A boy deposited half of his money in the savings bank; $\frac{1}{4}$ of the remainder he spent for clothes; and he had $3 remaining. How much had he at first?

61. Reduce .025 cwt. to lower denominations.

62. A man bought two city lots costing him $3500 and $4100 respectively. He sold them at a gain of 25%. What was the gain in dollars?

63. How many gallons of water will a tank contain that is 11 ft. long, $3\frac{1}{2}$ ft. wide, and 4 ft. deep?

64. A barn floor is 20 ft. wide and 45 ft. long. How much will it cost to cover it with plank 2 inches thick at $20 per thousand board feet?

65. Divide $\frac{3}{4}$ by $\frac{6}{9}$ of $\frac{3}{8}$.

66. John and James have together 165 acres of land, but James has twice as many acres as John. How many acres has each?

SUGGESTION. — 165 acres = twice John's + once John's or 3 times John's.

67. What fractional part of a day are 10 hours, 50 minutes, 40 seconds?

68. Divide nine ten thousandths by one hundred twenty-five thousandths.

69. What is the interest on $180 for 4 yr. 8 mo. at $5\frac{1}{2}$%?

70. I made $1.95 by selling 15 dozen eggs at $.31 per dozen. What was the cost of the eggs per dozen?

71. What is the gain per cent on flour that cost $4.50 per barrel if sold at $5 per barrel?

72. A father divided his farm of 202 A. 16 sq. rd. equally among his four sons. How many acres did each receive?

73. .21 of a mile is equal to how many feet?

74. An automobile that cost $2675 was sold at a loss of 28%. For how much was it sold?

75. What is the cost of 18 plank 20 ft. long, 12 in. wide, and 2 in. thick, at $20 per M?

76. $\frac{1}{2}$ of 7 is what part of 9?

77. $\frac{5}{6}$ of a farm is worth $7500. What is 20% of the farm worth?

78. What is the cost of a car load of bituminous coal weighing 84,000 pounds at $2.65 per ton?

79. A farm in the form of a rectangle containing 120 acres is 60 rods wide. How long is it?

80. Express decimally the quotient of $\frac{7}{8} \div .35$.

81. If $\frac{1}{5}$ of a ton of hay is worth $12, how much are 33,000 pounds of hay worth?

82. What is the value of a pile of 4-foot wood 48 ft. long and 6 ft. high, at $4.50 per cord?

83. A dairyman owns a cow that averages 3 gal. 2 qt. 1 pt. of milk daily. If he sells the milk at $.06 per quart, how much will he realize from the cow during the month of May?

84. I can buy an automobile at one store for $3000, with discounts of 25%, 10%; or at another store for $3000 with only one discount of 35%. Which is the cheaper?

85. Mr. John Wiley bought from Charles Saybrook & Son, Grocers:

25 lb. sugar	@ \$0.06½
2 doz. bananas	@ \$0.20
2 lb. tea	@ \$0.60
8 lb. butter	@ \$0.29
4½ doz. eggs	@ \$0.27

Make out a receipted bill.

86. A car load of 20 head of cattle, averaging 1200 lb., was shipped to Chicago by an Iowa cattle man. The cattle were sold at 4½¢ per pound. The railroad charged \$7 per head freight. How much money was remitted to the cattle man?

87. Reduce 3436 inches to higher denominations.

88. A room is 14 ft. long, 12 ft. wide, and 10 ft. high. How much will it cost to plaster the walls and ceiling at \$.09 per square yard, allowing ⅛ of the whole surface for openings?

89. A owned ¾ of a store and sold ⅔ of his share for \$12,000. Find the value of the store.

90. If a merchant buys silk at \$1.25 a yard and sells it at \$1.75 a yard, what per cent does he gain?

91. A man bought a house for \$3760, and sold it at a loss of 37½ %. How much did he lose?

92. On a spelling test I spelled 468 words correctly and received 75 %. How many words were there in the test?

93. On another test I spelled correctly 380 words out of 400. What per cent of the words did I spell correctly?

94. How much commission, at 4 %, does a salesman receive for selling goods to the amount of \$16,500?

95. In a certain city there are 12,600 persons of school age, but only 9450 are enrolled in the school. What per cent of the persons of school age are there in the school?

96. A drover bought 250 sheep at $\$3\frac{4}{5}$ per head, and paid for $\frac{2}{5}$ of them at the time of purchase. He sold 60 sheep at $\$4\frac{1}{2}$ per head, and with the money made another payment. How much did he still owe?

97. An auctioneer sold \$16,200 worth of goods at 2 % commission. How much did the owner of the goods realize from the sale?

98. $\frac{2}{3}$ of $\frac{4}{5} \div .125$ equals what? Express this decimally.

99. What per cent is lost on books bought for \$560 and sold for \$490?

100. Find the net proceeds from the sale of 145 books, at \$2 each, on which a commission of $33\frac{1}{3}$ % is paid.

101. A man drew out 24 % of the money he had in bank, to invest in an automobile costing \$2880. How much money had he in the bank at first?

102. Find the area in acres of a street 7 miles long and 66 feet wide.

103. $66\frac{2}{3}$ % of \$2556 is $\frac{3}{7}$ of what number?

104. A mechanic's deposits in the savings bank are as follows: April, \$11; May, \$6.15; June, \$11; July, \$8.50; August, \$19.25; September, \$6; October, \$10; November, \$16; December, \$19.85; January, \$13; February, \$16; March, \$15. His daily wages are \$2.75, and he works 285 days in the year. Omitting interest, find his total deposit. What per cent of his wages does he save?

105. Frank Stewart borrows \$250 Sept. 15, 1906, at 6 % interest. Find the amount of the note if paid March 15, 1908.

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